KENDRIYA VIDYALAYA SANGATHAN
REGIONAL OFFICE BHOPAL

(2024-25)

QUESTION BANK

CLASS XII

MATHEMATICS

OUR PATRON

HONOURABLE DEPUTY COMMISSIONER
KVS RO BHOPAL REGION

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Chapter 1: Relation and function

Relation LEVEL I

1. A function $f(x):R_+ \longrightarrow R_+$ defined as $f(x)=x^2+1$ check whether it is one one ,and onto
2. Show that the relation R in the set N given by R ={(a , b)| a is divisible by b ,a ,b $\in$ N} is reflexive and transitive but not symmetric.
3. Let R be the relation in the set N given by R = {(a ,b)| a > b} Show that the relation is neither reflexive nor symmetric but transitive.
4. Let R be the relation on R defined as (a , b) $\in$ R iff 1+ ab > 0 $\forall$ a,b $\in$ R.
   (a) Show that R is symmetric.
   (b) Show that R is reflexive.
   (c) Show that R is not transitive.
5. Check whether the relation R is reflexive, symmetric and transitive.
   R = { (x ,y)| x – 3y = 0} on A ={1,2,3……….13,14}.
6. 6 Let R be the relation in the set N given by R = {(a,b)| a = b – 2 , b > 6}
   Whether the relation is reflexive or not ? justify your answer.

Relation LEVEL II

1. A relation R on set A=$\{-4,-3,-2,-1,0,1,2,3,4\}$ be defined as
   R=$\{(x,y): x+y \text{ is an integer divisible by } 2\}$ show that R is an equivalence relation.Also write the equivalence class of $\{2\}$

2.Let N be the set of all natural numbers & R be the relation on N × N defined by
{ (a , b) R (c , d) iff a + d = b + c}. Show that R is an equivalence relation.

1. Show that the relation R in the set A of all polygons as:
   R ={(P_1,P_2), P_1& P_2 have the same number of sides} is an equivalence relation. What is the set of all elements in A related to the right triangle T with sides 3,4 & 5 ?
2. Show that the relation R on A ,A = { x| x $\in$ Z , 0 ≤ x ≤ 12 } ,
   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.
3. Let N be the set of all natural numbers & R be the relation on N × N defined by
   { (a , b) R (c , d) iff a + d = b + c}. Show that R is an equivalence relation.
4. Let A = Set of all triangles in a plane and R is defined by R={(T_1,T_2) : T_1,T_2 $\in$ A & T_1~T_2 }
   Show that the R is equivalence relation. Consider the right angled Δs, T_1 with size 3,4,5;
   T_2 with size 5, 12,13; T_3 with side 6, 8, 10; Which of the pairs are related?

5. Show that the relation R on A ,A = { x| x $\in$ Ƶ , 0 ≤ x ≤ 12 } ,

   R = {(a ,b): |a - b| is multiple of 3.} is an equivalence relation.

Function LEVEL I

1 Let A = {-1, 0, 1} and B = {0 , 1}. State whether the function f : A → B defined by f(x) = x^2 is bijective

2 Show that the function f: R→R defined by f(x) = x^2 is neither one-one nor onto.

3 Show that the function f: N→N given by f(x) = 2x is one-one but not onto.

4 Show that the signum function f: R→R given by: $f(x) = \begin{cases} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{cases}$

   is neither one-one nor onto.

5 Let A = {-1, 0, 1} and B = {0 , 1}. State whether the function f : A → B defined by f(x) = x^2 is bijective .

Function LEVEL II

1. Let A = {1,2,3}, B = {4,5,6,7} and let f = {(1 , 4),(2 , 5), (3 , 6)} be a function from A to B. State whether f is one-one or not.
2. Show that the function f: R→R defined by f(x) = $\frac{2x-1}{3}$. x$\in$ R is one- one & onto function.
3. Consider a function f :R_+→[-5, ∞) defined f(x) = $9x^2$ + 6x - 5. Show that f is one- one & onto

4. Show that function f : R→R defined by f(x)= 7 - $2x^3$ for all x $\in$ R is bijective.

5. If f: R→R is defined by f(x) = $\frac{3x+5}{2}$. Show that f is one- one

Chapter 2: INVERSE TRIGONOMETRIC FUNCTIONS

LEVEL I -Multiple Choice Questions (For 1 Mark)

(Q1) Principal Value of $\sin^{-1}\left(-\frac{1}{2}\right)$

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $-\frac{\pi}{6}$

(Q2) Principal Value of $\cos^{-1}\left(\frac{1}{2}\right)$

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{3}$ (c) $\frac{5\pi}{6}$ (d) $-\frac{5\pi}{6}$

(Q3) The Value of $\sin^{-1}\left(cos\ \frac{13\pi}{5}\right)$

(a) $-\frac{3\pi}{5}$ (b) $-\frac{\pi}{10}$ (c) $\frac{3\pi}{5}$ (d) $\frac{\pi}{10}$

Q4) $sin[\frac{\pi}{2} - \sin^{-1}\left(-\frac{1}{2}\right)]$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) -1 (d) 1

(Q5) The Value of $\tan^{-1}\left(sin\ (-\frac{\pi}{5})\right)$

(a) -1 (b) 1 (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{4}$

(Q6) The Principal Value of $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

(a) $\frac{\pi}{12}$ (b) π (c) $\frac{5\pi}{6}$ (d) $\frac{\pi}{6}$

(Q7) What is the domain of $\cos^{-1}(2x - 3)$

(a) $[-1, 1]$ (b) $(-1, 1)$ (c) $[1, 2\ ]$ (d) $(1,\ 2)$

(Q8) The Value of $\sin^{-1}\left(sin\ \frac{6\pi}{7}\right)$

(a) $-\frac{6\pi}{7}$ (b) $-\frac{\pi}{7}$ (c) $\frac{6\pi}{7}$ (d) $\frac{\pi}{7}$

(Q9) $sin[\frac{\pi}{3} + \sin^{-1}\left(\frac{1}{2}\right)]$ is equal to

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) 1

(Q10) The Value of $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$ is

(a) $\frac{\pi}{3}$ (b) $-\frac{\pi}{2}$ (c) $\frac{\pi}{2}$ (d) $-\frac{\pi}{6}$

Answers

1. (d) $-\frac{\pi}{6}$	2. (a) $\frac{\pi}{3}$	3. (b) $-\frac{\pi}{10}$	4. (d) 1	5. (d) $-\frac{\pi}{4}$
6. (a) $\frac{\pi}{12}$	7. ((c) $[1, 2\ ]$	8. d) $\frac{\pi}{7}$	9. (d) 1	10. (b) $-\frac{\pi}{2}$

LEVEL II –Very Short Answer type Questions (For 2 Marks)

(Q1) Evaluate: $3\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)+\cos^{-1}(0)$

(Q2) Evaluate $\cos^{-1}\left(cos\left(\frac{-7\pi}{3}\right)\right)$.

(Q3) Evaluate: $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)+\tan^{-1}1$

(Q4) Find the domain of: $\sin^{-1}(x^2-4)$.

(Q5) Draw the graph of $\cos^{-1}x$, where $x\in[-1,0]$. Also write its range.

(Q6) Draw the graph of $\sin^{-1}x$, where $x\in\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{1}}\right]$. Also write its range.

(Q7) Find the principal value of $\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)$.

(Q8) Evaluate: $\sin^{-1}\left(sin\frac{3\pi}{4}\right)+\cos^{-1}\left(cos\frac{3\pi}{4}\right)+\tan^{-1}1$

(Q9) If $\sin^{-1}x=\frac{\pi}{2}-\sec^{-1}\frac{4}{3}$, then find the value of x .

(Q10) Find the l value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$.

Answers

1. $\frac{19\pi}{12}$	2. $\frac{\pi}{3}$	3. $\frac{3\pi}{4}$	4. $[-\sqrt{5},-\sqrt{3}]\cup[\sqrt{3},5]$	5. $\left[-\frac{\pi}{2},\pi\right]$
6. $\left[-\frac{\pi}{4},\frac{\pi}{4}\right]$	7. $-\frac{\pi}{3}$	8. $\frac{5\pi}{4}$	9. $\frac{3}{4}$	10. $\frac{\pi}{4}$

* LEVEL III –Short Answer type Questions (For 3 Marks)

(Q1) Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

(Q2) Solve $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\frac{8}{31}$

Ans x =¼

(Q3) Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

(Q4) Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right)=\frac{4-\sqrt{7}}{3}$

(Q5) Prove that $\tan^{-1}\left(\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$

ASSERTION-REASON BASED QUESTIONS

Directions: Each of these questions contains two statements, A= Assertion and R= Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the choices (a),(b),(c) and (d) given below-

(a) A is correct, R is correct, R is a correct explanation for A.

(b) A is correct, R is correct, R is not a correct explanation for A.

(c) A is correct, R is incorrect.

(d) A is incorrect, R is correct.

(Q1) A: $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)=\frac{5\pi}{6}$

R: $\cos^{-1}(\cos x)=x$, when $\mathbf{0} \leq \mathbf{x} \leq \boldsymbol{\pi}$

Ans-(a)

(Q2) A : The value of $\mathbf{sin}\left[\mathbf{sin}^{-1}(\frac{4}{5})+\mathbf{sec}^{-1}\left(\frac{5}{4}\right)\right]=\mathbf{1}$ 1

R :$\mathbf{sin}^{-1}\boldsymbol{x}+\mathbf{cos}^{-1}\boldsymbol{x}=\frac{\pi}{2}$; $\mathbf{0}<x<\frac{\pi}{2}$ $\boldsymbol{and}$ $\mathbf{cos}^{-1}\boldsymbol{y}=\mathbf{sec}^{-1}(-\boldsymbol{y})$

Ans-(c)

(Q3) A : 2 $\mathbf{tan}^{-1}(\frac{1}{2})+\ \mathbf{tan}^{-1}(\frac{1}{7})=\ \mathbf{tan}^{-1}(\frac{32}{17})$ 1

R : 2 $\mathbf{tan}^{-1}\boldsymbol{x}=\mathbf{tan}^{-1}\left(\frac{2x}{1-x^2}\right)$ $\boldsymbol{where}\ -\mathbf{1}<x<1.$

$$\boldsymbol{and}\ \mathbf{tan}^{-1}\boldsymbol{x}+\mathbf{tan}^{-1}\boldsymbol{y}=\mathbf{tan}^{-1}\left(\frac{\boldsymbol{x}+\boldsymbol{y}}{\mathbf{1}-\boldsymbol{xy}}\right)\ \boldsymbol{where}\ \boldsymbol{xy}<1$$

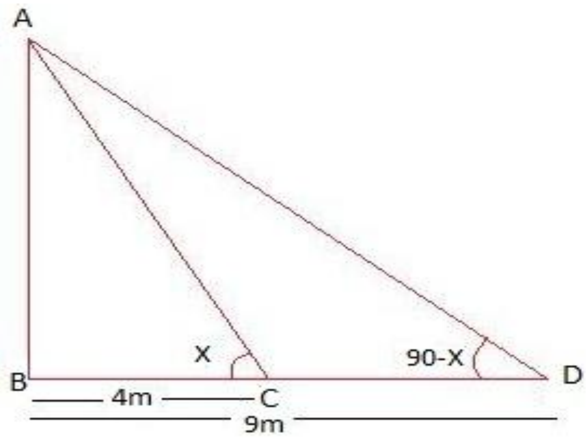
Ans-(d)

CASE STUDY QUESTIONS

CASE STUDY 1

The angles of elevations of the top of a building AB from two points C and D at a distance of 4 m and 9 m respectively from the base of the building and in the same straight line with it are complementary.

Look at the figure given and based on the above information answer the following:



(Q1) Measure of ∠ACB = 1

(a) $\sin^{-1}(\frac{3}{\sqrt{13}})$ (b) $\sin^{-1}(\frac{2}{\sqrt{13}})$ (c) $\tan^{-1}(\frac{2}{3})$ (d) $\sin^{-1}(\frac{\sqrt{13}}{3})$

(Q2) Following is false 1

(a) $\sin^{-1}(\frac{3}{\sqrt{13}})+\sec^{-1}(\frac{\sqrt{13}}{2}) = \frac{\pi}{2}$ (b) $\frac{\pi}{2} - X = \tan^{-1}(\frac{3}{2})$

(c) $\frac{\pi}{2} - X = \tan^{-1}(\frac{2}{3})$ (d) $\sin^{-1}(\frac{3}{\sqrt{13}}) = \text{cosec}^{-1}(\frac{\sqrt{13}}{3})$

(Q3) Measure of ∠BAC= 1

(a) $\sin^{-1}(\frac{3}{\sqrt{13}})$ (b) $\cos^{-1}(\frac{2}{\sqrt{13}})$ (c) $\tan^{-1}(\frac{3}{2})$ (d) $\cot^{-1}\left(\frac{3}{2}\right)$

(Q4) If height of building AB is h then following is true 1

(a) $X = \tan^{-1}\frac{h}{4}$ and $\frac{\pi}{2} - X = \tan^{-1}\frac{h}{9}$ then h = 36 m

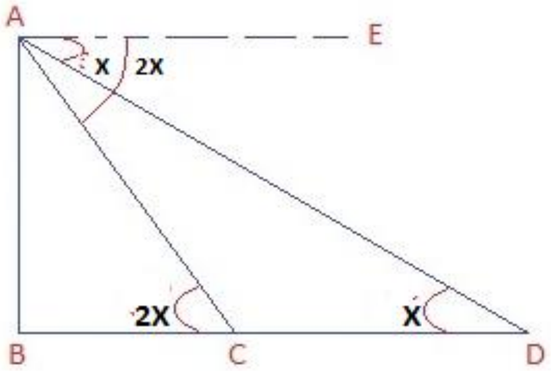
(b) $X = \cot^{-1}\frac{h}{4}$ and $\frac{\pi}{2} - X = \tan^{-1}\frac{h}{9}$ then h = 6 m

(c) $X = \tan^{-1}\frac{h}{4}$ and $\frac{\pi}{2} - X = \tan^{-1}\frac{h}{9}$ then h = 6 m

(d) $X = \tan^{-1}\frac{h}{4}$ and $\frac{\pi}{2} - X = \cot^{-1}\frac{h}{9}$ then h = 6 m

CASE STUDY 2

A straight highway leads to the foot B of a tower AB. A man standing at the top of the tower observes a car at point D at an angle of depression X, which is approaching the foot of the tower with a uniform speed 60 Km/hr. Six seconds later, the angle of depression of the car at point C is found to be 2X. The distance of point C from foot of tower B is 60 m.



Based on the above information answer the following:

(Q5) Measure of ∠ADB = X= 1

(a) $\sin^{-1}(\frac{1}{2})$ (b) $\sin^{-1}(\frac{4}{3})$ (c) $\tan^{-1}(\frac{1}{2})$ (d) $\tan^{-1}(\frac{4}{3})$

(Q6) Measure of ∠BCA= 1

(a) $\sin^{-1}(\frac{3}{4})$ (b) $\cos^{-1}(\frac{3}{5})$ (c) $\tan^{-1}(\frac{3}{2})$ (d) $\cot^{-1}(\frac{3}{5})$

(Q7) Following is true 1

(a) $\frac{1}{2}\tan^{-1}\frac{4}{3} = \tan^{-1}\frac{1}{2}$ (b) $\tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{2} = \tan^{-1}\frac{5}{2}$

(c) $\frac{1}{2}\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{4}{3}$ (d) $\tan^{-1}\frac{4}{3} - \tan^{-1}\frac{1}{2} = \tan^{-1}\frac{11}{2}$

(Q8) Height of the tower AB is 1

(a) 100 m (b) 60 m (c) 160 m (d) 80 m

ANSWERS

CASE STUDY 1	Q1	Q2	Q3	Q4
Answers	(a)	(b)	(d)	(c)
CASE STUDY 2	Q5	Q6	Q7	Q8
Answers	(c)	(b)	(a)	(d)

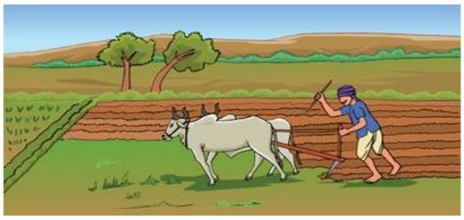
Chapter 3: MATRICES

Q	QUESTIONS	MARKS
1	If A is a square matrix such that $A^2 = A$, then $(I + A)^2 - 3A$ is (*a*) I (*b*) $2A$ (c) $3I$ (*d*) A	1
2	The diagonal elements of a skew symmetric matrix are (*a*) all zeroes (*b*) are all equal to some scalar $k(\neq 0)$ (*c*) can be any number (*d*) none of these	1
3	If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A'$ then (*a*) $x = 0, y = 5$ (*b*) $x = y$ (*c*) $x + y = 5$ (*d*) $x - y = 5$	1
4	If $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$, then write the value of x and y. (a) x = 3, y = 3 (b) x = 3, y = 2 (c) x = 2, y = 2 (d) x = 2, y = 3	1
5	A is a skew-symmetric matrix and a matrix B such that B'AB is defined, then B'AB is a: (a) symmetric matrix (b) skew-symmetric matrix (c) Diagonal matrix (d) upper triangular symmetric	1
6	.If A is a symmetric matrix, then A^3 is: (a) Symmetric Matrix (b) Skew Symmetric Matrix (c) Identity matrix (d) Row Matrix	1
7	If F(x) = $\begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ prove that F(x) F(y) = F(x+y)	1
8	If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then write the value of k. (a) 17 (b) -17 (c) 13 (d) -13	1
9	If $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$, find the value of *x*. (a) 1 (b) 2 (c) 3 (d) 4	1
10	The matrix A = $\begin{bmatrix} 0 & 2b & -2 \\ 3 & 1 & 3 \\ 3a & 3 & -1 \end{bmatrix}$ is a symmetric matrix. Then the value of a and b respectively are:	1

	(a) $\frac{-2}{3}, \frac{3}{2}$ (b) $\frac{-1}{2}, \frac{1}{2}$ (c) -2, 2 (d) $\frac{3}{2}, \frac{1}{2}$	
11.	If $\begin{bmatrix} 2 & 0 \\ 5 & 4 \end{bmatrix} = \mathrm{P} + \mathrm{Q}$, where P is symmetric and Q is a skew symmetric matrix, then Q is equal to (a) $\begin{bmatrix} 2 & 5/2 \\ 5/2 & 4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & -5/2 \\ 5/2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -5/2 \\ 5/2 & 1 \end{bmatrix}$	1
12	If A is 2×3 matrix such that AB and AB$'$ both are defined, then find the order of the matrix B (a) 2×3 (b) 3×3 (c) 2×2 (d) Not defined	1
13	.If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) of x is/are (a) 1 (b) $\sqrt{3}$ (c) $-\sqrt{3}$ (d) $\pm\sqrt{3}$	1
14	If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is (a) 1 (b) -1 (c) 2 (d) 0	
	For Q15 and Q16, a statement of assertion (A) is followed by a statement of reason (R). Choose the correct answer out of the following choices. (a) Both A and R are true and R is the correct explanation of A. (b) Both A and R are true but R is not the correct explanation of A. (c) A is true but R is false. (d) A is false but R is true.	
15	Assertion (A): The matrix A = $\begin{bmatrix} 0 & -1 & -2 \\ 1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix}$ is a skew symmetric matrix. Reason (R): For the given matrix A we have A' = A.	1
16	**1.** Assertion (A): Let A = $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 4 & 7 \end{bmatrix}$ and B = $\begin{bmatrix} 4 & 3 & 6 \\ 7 & 8 & 9 \\ 5 & 1 & 2 \end{bmatrix}$, then the product of the matrices A and B is not defined. Reason (R): The number of rows in B is not equal to number of columns in A.	1
17	Find the value of *a*, *b*, *c* and *d* from the equation: $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$	2

18	Find X and Y, if X + Y = $\begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}$ and X – Y = $\begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}$.	2
19	Find the values of *x*, *y* and *z*, if $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$	2
20	Solve the system of the following equations: $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=2, \quad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=5, \quad \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=-4$	2
21	If A = $\begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$, B = $\begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB and hence solve x – 2y = 10, 2x + y + 3z = 8, - 2y + z = 7	2
22	If A = $\begin{bmatrix} -3 & -2 & -4 \\ 2 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, and B = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find AB and use it to solve the following system of equations: x – 2y = 3, 2x – y – z = 2, – 2y + z = 3.	2
23	If A = $\begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, find A^{-1}. Using A^{-1}, solve the system of linear equations: x – 2y = 10, 2x – y – z = 8, – 2y + z = 7.	2
24	Find the values of *x* and *y* from the following equation: $2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$	2
25	If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$	5
26	If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that I + A = (I – A)$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$	5
27.		5

	Express the matrix $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ as the sum of a symmetric and a skew symmetric matrix.	
28	Given A = $\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and B = $\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, verify that BA = 6I, how can we use the result to find the values of x, y, z from given equations x – y = 3, 2x + 3y + 4z = 17, y + 2z = 17	5
	<u>(Case Study Based Questions)</u> Questions 21 to 22 carry 4 marks each.	
29	To promote the usage of house toilets in villages, especially for women, are organisations tried to generate awareness among the villagers through (i) house calls (ii) letters, and (iii) announcements.  The cost for each mode per attempt is given below. (i) ₹ 50 (ii) ₹ 20 (iii) ₹ 40 The number of attempts made in villages X, Y, and Z is given below: $\begin{array}{cccc} & (i) & (ii) & (iii) \\ X & 400 & 300 & 100 \\ Y & 300 & 250 & 75 \\ Z & 500 & 400 & 150 \end{array}$ Also, the chance of making toilets corresponding to one attempt of given modes is: (i) 2% (ii) 4% (iii) 20% Let A, B, and C be the cost incurred by organisation in three villages respectively. Based on the above information answer the following questions: (i) Form a required matrix on the basis of the given information. [1] (ii) From a matrix, related to the number of toilets expected in villagers X, Y, and Z after the promotion campaign. [1] (iii) What is the total amount spent by the organisation in all three villages X, Y, and Z?	4

	[2] OR (iii) What is the total no.of toilets expected after the promotion campaign? [2]	
30	Two farmers Ankit and Girish cultivate only three varieties of pulses namely Urad, Masoor and Mung. The sale (in Rs.) of these varieties of pulses by both the farmers in the month of September and October are given by the following matrices A and B.  September sales (in Rs.) $A = \begin{pmatrix} Urad & Masoor & Mung \\ 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$ October sales (in Rs.) $B = \begin{pmatrix} Urad & Masoor & Mung \\ 5000 & 10000 & 6000 \\ 20000 & 30000 & 10000 \end{pmatrix} \begin{matrix} \\ Ankit \\ Girish \end{matrix}$ (i) Find the combined sales of Masoor in September and October, for farmer Girish. [1] (ii) Find the combined sales of Urad in September and October, for farmer Ankit. [1] (iii) Find a decrease in sales from September to October. [2] OR (iii) If both the farmers receive 2% profit on gross sales, then compute the profit for each farmer and for each variety sold in October. [2]	4

CHAPTER 4: DETERMINANT

MULTIPLE COICE QUESTIONS (1 MARK EACH)

1 If A is a Singular Matrix then A(adjA) is

(a) Scalar matrix (b) Null matrix (c) Identity matrix (d) None of these

2 If P is a square matrix of order 3,such that P(adjoint P) =10 I , then the determinant of adjoint P is equal to (a) 0 (b) 1 (c) 10 (d) None of these

3 The area of a triangle with vertices (-3,2), (5,4), (k,-6) is 42 sq units . What is the value of k?

(a) 6 (b) 5 (c) 7 (d) None of these

4. A system of equations is said to be inconsistent if the solution

(a) exists (b) is unique (c) does not exist (d) None of these

5. If A is a square matrix of order 3 and det A=7 what is the value of det (adjoint A) ?

(a) 39 (b) 49 (c) 30 (d) None of these

6 A square matrix is invertible if and only if

(a) A is not a non- singular matrix (b) A is a singular matrix

(c) A is a non-singular matrix (d) None of these

7 If |A| = |kA|, where A is a square matrix of order 2, then sum of all possible values of k is

(a) 1 (b) -1 (c) 2 (d) 0

8 If A is a square matrix of order 3 and |A| = 5, then |adj A| is

(a) 5 (b) 25 (c) 125 (d)1/5

9. The area of triangle with vertices (-3,0), (3,0) and (0,k) is 9 sq units. Then the value of k will be

(a) 9 (b) 3 (c) -9 (d) 6

10 If A is a square matrix of order 3 such that |A| = - 5 , then value of |−A| is

(a) 125 (b) - 125 (c) 5 (d) – 5

11 If A is a square matrix of order 3 such that $|adjA|$=64, then what is the value of $|A|$

(a)64 (b) 8 (c) −8 (d) ±8

12 If A (3,4),B(−7,2),$C(x,y)$ are collinear, then which of the following is true?

(a) x+5y+17=0 (b) x+5y+13=0

(c) x−5y+17=0 (d) x−5y−17=0

13 If A is a square matrix such that A^2= I, then A^{-1} is equal to:

a) 2A (b) O (c) A (d) A+I

14 If A and B are square matrix of order of 3 such that $|A|$= −1 and $|B|$=3 then what is the value of $|3AB|$?

a) −9 (b) −27 (c) −81 (d) 81

15 If A=$\begin{bmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{bmatrix}$ then the value of $|adj(A)|$ is

a) x^3 (b) x^6 (c) x^9 (d) x^{27}

16 If A is a square matrix such that square of A =I then inverse of A is

(a) A (b) 2A (c) A/2 (d) None of these

17 If A is a square matrix of order 3 and | A |=7 what is the value of | adj A| ?

(a) 39 (b) 49 (c) 30 (d) None of these

18 The sum of the products of elements of any row with the co-factors of corresponding elements is equal

(a) Adjoint of the matrix (b) 0 (c)1 (d) Value of the determinant

ANSWER KEY

1. (b) Null matrix 2. (c) 10 3. (c) 7 4. (c) does not exist 5. (b) 49 6. (c) A is a non-singular matrix. 7. (d) 0 8 (b) 25 9. (b) 3 10. (c) 5 11. (d) ±8 12. (c) $x-5y+17=0$ 13. (c) A

14. (c) −81 15. (b) x^6 16. (a) A 17. (b) 49 18. (d) Value of the determinant

ASSERTION-REASON BASED QUESTIONS (1 MARK EACH)

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as

(i) Both A and R are true and R is the correct explanation of A.

(ii) Both A and R are true and R is not the correct explanation of A.

(iii) A is true but R is false.

(iv) A is false but R is true.

1 Assertion (A) : adj A is a non- singular matrix

Reason (R) : A is non – singular matrix

2. Assertion (A) : For two matrices A and B of order 3,|A| =3,|B| = -4 then |2AB| = -96

Reason (R) : For a matrix A of order n and a scalar k, det (k A) = k^n (det A)

3. For A and B square matrices of same order, choose appropriate option

Assertion (A): $(A + B)^2 \neq A^2 + 2AB + B^2$

Reason (R): Generally, AB ≠ BA

4. Assertion (A) : adj A is a non- singular matrix

Reason (R) : A is non – singular matrix

5 Assertion (A) : A^{-1} exists

Reason (R) : | A | = 0

6 Assertion (A) : | Q |= O

Reason (R) : Determinant of skew symmetric matrix is O

ANSWER KEY

1. (i) 2. (i) 3. (i) 4. (i) 5. (iii) 6. (iii)

VERY SHORT ANSWER TYPE QUESTIONS (2 MARKS EACH)

1. Find the value of p, so that the area of a triangle is 16 sq. units and vertices are$(0,p)$,(0,4) and (−2,0).

2. For what value of x, the matrix $\begin{bmatrix} 5-x & x+1 \\ 2 & 4 \end{bmatrix}$is singular?

3. Find the equation of line joining (1,2) and (3,6) using Determinants.

4 Given A=$\begin{bmatrix} 2 & -3 \\ -4 & 7 \end{bmatrix}$, then compute A^{-1}.

5 If A is a square matrix of order 3, $|A'|$ = - 3, then find the value of $|AA'|$.

6 If x $\in$ N and $\begin{vmatrix} x+3 & -2 \\ -3x & 2x \end{vmatrix} = 8$, then find the value of x.

7 If $\begin{vmatrix} 2 & 3 & 2 \\ x & x & x \\ 4 & 9 & 1 \end{vmatrix}$ + 3 = 0, then find the value of x.

8. Evaluate the determinant $\begin{bmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{bmatrix}$

9 Find values of *x* for which $\begin{bmatrix} 3 & x \\ x & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$

10 Find adjA for A = $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

ANSWER KEY

1. P=$\mp$12 2. x=3 3. $-4x+2y=0$ or $2x-y=0$ is required line. 4. $A^{-1}=\frac{adjA}{|A|}=\begin{bmatrix} 7/2 & 3/2 \\ 2 & 1 \end{bmatrix}$

5. 9 6. 2 7. –1 8. -52 9. x = $\pm 2\sqrt{2}$ 10. $\begin{bmatrix} 4 & -3 \\ -1 & 2 \end{bmatrix}$

SHORT ANSWER TYPE QUESTIONS (3 MARKS EACH)

1 Find the value of y if $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix}$ = $\begin{vmatrix} 2y & 4 \\ 6 & y \end{vmatrix}$.

2 A is an invertible matrix of order 3x3 and |A | =7, then find | A^{-1} |.

3. Verify A(adj. A) =(adj. A)A = | A| I for A= $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

4. Solve using matrix method 2x-y = 1, 3x+2y=5

5 If A is a 3×3 invertible matrix, then what will be the value of k if $|A^{-1}| = |A|^k$

6 If A is a square matrix of order 3, |A'| = - 3, then find the value of |AA'|.

7 Show that the points (a, 0) , (0, b) and (1,1) are collinear if a + b = ab

8 Find the equation of the line joining P(4,0) and Q(0,2) using determinants and find λ if R(λ,0) is a point such that area of triangle PQR is 4 sq units.

9 If A = $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$, then show that | 3 A | = 27 | A |

10 Find values of *k* if area of triangle is 4 sq. units and vertices are

(i) (*k*, 0), (4, 0), (0, 2) (ii) (–2, 0), (0, 4), (0, *k*) by using determinant

11 Find minors and cofactors of the elements of the determinant

$\begin{bmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{bmatrix}$ and verify that $a_{11} A_{31} + a_{12} A_{32} + a_{13} A_{33} = 0$

12 If A = $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ $and\ B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then verify that $(AB)^{-1} = B^{-1}A^{-1}$.

13 Show that the matrix A = $\begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$ satisfies the equation $A^2 - 4A + I = O$,

where I is 2 × 2 identity matrix and O is 2 × 2 zero matrix. Using this equation, find A^{-1}.

14 Solve the system of equations by matrix method

$2x + 5y = 1$

$3x + 2y = 7$

ANSWER KEY

1. y = $\pm\sqrt{3}$ 2. 1/7 4. x=1 , y =1 5. -1 6. 9 8. 3x – y = 0; k = ±2, 10. (i) 0,8 (ii) 0,8 13. 13 . $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ 14. *x* = 3, *y* = – 1

LONG ANSWER TYPE QUESTIONS (5 MARKS EACH)

1. If A = $\begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 2 \\ 7 & 3 & -3 \end{bmatrix}$ then find A^{-1}. Hence solve the system of equations

3x+4y+7z+14=0, 2x-y+3z+4=0, x+2y-3z=0

2. Solve system of equations

x-y+2z=7, 2x-y+3z =12, 3x+2y-z =5 using matrix method

3. Solve the system of the following equations: $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=2\,,\frac{4}{x}-\frac{6}{y}+\frac{5}{z}=5\,,\frac{6}{x}+\frac{9}{y}-\frac{20}{z}=-4\,.$

4 If A = $\begin{bmatrix}-3 & -2 & -4\\ 2 & 1 & 2\\ 2 & 1 & 3\end{bmatrix}$, and B = $\begin{bmatrix}1 & 2 & 0\\ -2 & -1 & -2\\ 0 & -1 & 0\end{bmatrix}$, then find AB and use it to solve the following system of equations: x – 2y = 3, 2x – y – z = 2, – 2y + z = 3.

5 If A = $\begin{bmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{bmatrix}$ then verify that A *adj* A = |A| I. Also find A^{-1}.

6 For the matrix A = $\begin{bmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{bmatrix}$ Show that $A^3 - 6A^2 + 5A + 11\,I = O$. Hence, find A^{-1}.

7 The sum of three numbers is 6. If we multiply third number by 3 and add second number to it, we get 11. By adding first and third numbers, we get double of the second number. Represent it algebraically and find the numbers using matrix method.

8 If A = $\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

9 Use product $\begin{bmatrix}1 & -1 & 2\\ 0 & 2 & -3\\ 3 & -2 & 4\end{bmatrix}\begin{bmatrix}-2 & 0 & 1\\ 9 & 2 & -3\\ 6 & 1 & -2\end{bmatrix}$ to solve the system of equations

$x - y + 2z = 1$

$2y - 3z = 1$

$3x - 2y + 4z = 2$

10 Let A = $\begin{bmatrix}1 & -2 & 1\\ -2 & 3 & 1\\ 1 & 1 & 5\end{bmatrix}$. Verify that

(i) $[adj\,A]^{-1} = adj\,(A^{-1})$ (ii) $(A^{-1})^{-1} = A$

ANSWER KEY

1. x= -1, y = -1, z = -1 2. x=2, y=1, z=3 3. x = 2, y = –3, z = 5 4. AB = I; **x=1,y=−1,z= 1**, 5. $\begin{bmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{bmatrix}$ 6. $\frac{1}{11}\begin{bmatrix}-3 & 4 & 5\\ 9 & -1 & -4\\ 5 & -3 & -1\end{bmatrix}$ 7. $x = 1$, $y = 2$, $z = 3$ 8. $\begin{bmatrix}0 & 1 & -2\\ -2 & 9 & -23\\ -1 & 5 & -13\end{bmatrix}$,x=1 , y=2 , z=3

9. $x = 0$, $y = 5$ and $z = 3$.

CASE STUDY BASED QUESTIONS (4 – MARKS EACH)

1. A school wants to award its students for the values of honesty, regularity and hard work with a total cash award of Rs 6000. Three times the award money for hard work added to that given for honesty amounts to Rs 11000. The award money given for honesty and hard work together is double the one given for regularity.

   Based on the above information, answer the following questions:

   (i) If Rs x is awarded to honesty, Rs y to regularity and Rs z awarded to hard work, then what is the matrix equation representing the above situation ?

   (ii) What is the value of |adj A| ?

   (iii) What are the values of x, y, z respectively in this case ?

   (iv) What is the value of $|A^{-1}|$?

   (v) What is the value of A(Adj A) ?

2 Two school KV-1 and KV-2 want to award their selected students on the values of sincerity, truthfulness and helpfulness. KV-1 wants to award Rs x each, Rs y each and Rs z each for the three respective values to 3, 2 and 1 students respectively with a total award money of Rs 1600. KV-2 wants to spend Rs 2300 to award its 4, 1, 3 students on the respective values (by giving the same amount of the three values as before). The total amount of the award for one prize on each is Rs 900. Answer the following questions using matrix:

   (i) Determine x + y + z

   (ii) Calculate the value of 2x + y +3 z.

   (iii) What is the value of y?

   (iv) Find the value of 2x + 5y.

   (v) Determine y – x.

3 On her birth day, Seema decided to donate some money to children of an orphanage home. If there were 8 children less, everyone would have got Rs.10 more. However, if there were 16 children more, everyone would have got Rs. 10 less. Let the number of children be x and the amount distributed by Seema for one child be y (in Rs.)

Based on the information given above, answer the following questions:

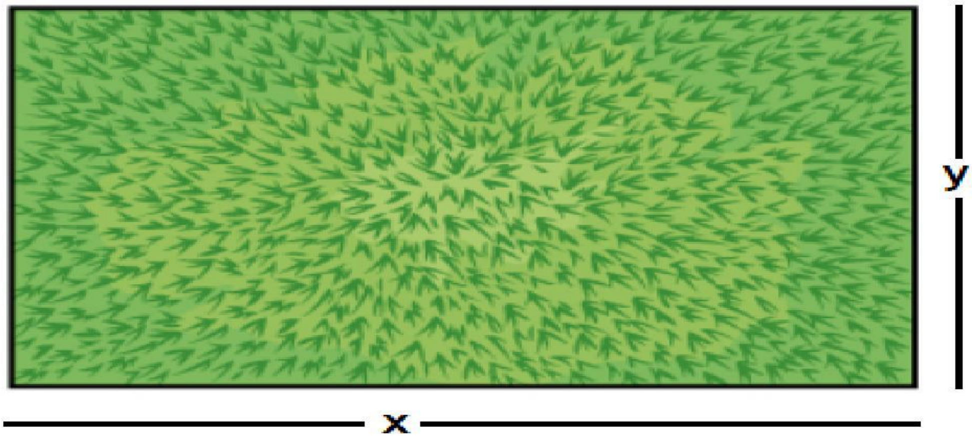
(i). write above situation in equations in terms x and y

(ii) Which of the following matrix equations represent the information given above?

(iii)find number of children who got some money from Seema.

(iv) How much amount is given to each child by Seema?

4 Manjit wants to donate a rectangular plot of land for a school in his village. When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.



Based on the information given above, answer the following questions :

(i) The equations in terms of x and y are

(a) x - y = 50, 2x - y =550

(b) x - y = 50, 2x + y =550

(c) x +y = 50, 2x + y =550

(d) x + y = 50, 2x - y =550

(ii) Which of the following matrix equation represents the information given above?

(a) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(iii) The value of x (length of rectangular field), is

(a) 150 m (b) 400 m (c) 200 m (d) 320 m

(iv) The value of y (breadth of rectangular field), is

(a) 150 m (b) 200 m (c) 430 m (d) 350 m

(v) How much is the area of rectangular field?

(a) 60000 Sq. m (b) 30000 Sq. m (c) 30000 m (d) 3000 m

5 To promote the usage of house toilets in villages especially for women, an organisation tried to generate awareness among the villagers through (i) house calls (ii) letters and (iii) announcements

The cost for each mode per attempt is (i) Rs 50 (ii) Rs 20 (iii)Rs40 respectively

The number of attempts made in the villages X, Y and Z are given below:

	(i)	(ii)	(iii)
X	400	300	100
Y	300	250	75
Z	500	400	150

Also the chance of making of toilets corresponding to one attempt of given modes is:

(i) 2% (ii) 4% (iii) 20%

Let A, B, C be the cost incurred by organization in three villages respectively.

Based on the above information answer the following questions

(A) Form a required matrix on the basis of the given information.

(B) Form a matrix, related to the number of toilets expected in villagers X, Y, Z after the promotion campaign.

(C) What is total amount spent by the organisation in all three villages X, Y and Z

OR

What are the total number of toilets expected after promotion campaign?

ANSWER KEY

1. (i) $\begin{bmatrix} \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} & \mathbf{3} \\ \mathbf{1} & \mathbf{-2} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix} = \begin{bmatrix} \mathbf{6000} \\ \mathbf{11000} \\ \mathbf{0} \end{bmatrix}$ (ii). 36, (iii). (500, 2000, 3500), (iv). $\mathbf{1/6}$, (v). 6I,

2. (i). 900 (ii). 1900 (iii). 300 (iv). 1900 (v). 100

3. (i). 5x-4y = 40 ,5x-8y = -80

(ii). $\begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

(iii). 32

(iv) . Rs.30

4. (i) (b) x - y = 50, 2x + y =550

(ii) (a) $\begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 50 \\ 550 \end{bmatrix}$

(iii) (c) 200 m

(iv) (a) 150 m

(v) (b) 30000 Sq. m

5. A- Rs A, Rs B and Rs C are the cost incurred by the organisation for villages X, Y, Z respectively, therefore matrix equation will be

$$\begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 50 \\ 20 \\ 40 \end{bmatrix} = \begin{bmatrix} \mathrm{A} \\ \mathrm{B} \\ \mathrm{C} \end{bmatrix}$$

B- Let number of toilets expected in villagers X, Y, Z be x, y, z respectively

Therefore required matrix is

$$\begin{bmatrix} \mathrm{x} \\ \mathrm{y} \\ \mathrm{z} \end{bmatrix} = \begin{bmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{bmatrix} \begin{bmatrix} 2/100 \\ 4/100 \\ 20/100 \end{bmatrix}$$

C- Total money spent = 30000 + 23000 + 39000 = 92000 Rs

OR

Total number of toilets expected in 3 villages are = 40 + 31 + 56 = 127

Chapter 5 : DIFFERENTIABILITY

Multiple Choice questions:

Q1 If y = sinx, then $\frac{d^2y}{dx^2}$ $at\ x = \frac{\pi}{2}$ is equal to :

(a) 2 (b) 0 (c) -1 (d) 1

Q2 If y = log tan $\sqrt{x}$ then the value of $\frac{dy}{dx}$ is

(a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{\sec 2\sqrt{x}}{\sqrt{x}\tan x}$ (c) 2 $\sec^2\sqrt{x}$ (d) $\frac{\sec 2\sqrt{x}}{2\sqrt{x}\tan x}$

Q3 If y = ($\cos x^2$)^2 then $\frac{dy}{dx}$ is

(a) -4 x sin $2x^2$ (b) – x sin x^2 (c) -2 x sin $2x^2$ (d) - x cos 2 x^2

Q4 If y = $\cot^{-1}(x^2)$ then the value of $\frac{dy}{dx}$ is

(a) $\frac{2x}{1+x4}$ (b) $\frac{2x}{\sqrt{1+4x}}$ (c) $\frac{-2x}{1+x4}$ (d) $\frac{-2x}{\sqrt{1+x2}}$

Q5 The differential coefficient of *f(logx)* where *f(x) = logx* is

(a) $\frac{1}{xlogx}$ (b) $\frac{logx}{x}$(c)$\frac{x}{logx}$ (d) None of these

Q6 If y = x^x, then $\frac{dy}{dx}$ is

(a) xx^{-1} (b) x^x(1 + log*x)* (c) x^x*logx* (d)None of these

Q7 The differential coefficient of x^9 w.r.t. x^3 is

(a) $9x^8$ (b)$3x^6$ (c)6 x^6 (d) None of these

Q8 If f(x) = $\begin{cases} 3x-5 & x \le 5 \\ 2k & x > 5 \end{cases}$ is continuous at x=5 then k is

(a)5 (b) 10 (c) 15 (d) $\frac{-2}{7}$

Q 9. The function f(x) = [x]is continuous at

(a) 4 (b) -2 (c) 1 (d) 1.5

Q 10 If x = t^2 and y = t^3 then $\frac{d^2y}{dx^2}$ is equal to

(a) $\frac{3}{2}$ (b) $\frac{3}{4t}$ (c) $\frac{3}{2t}$ (d) $\frac{3t}{2}$

ANSWERS:

Q1 (c) Q2 (d) Q3 (c) Q4 (c) Q5 (a) Q6 (b) Q7 (b) Q8 (a) Q9 (d) Q10 ((b)

Very Short Questions

Q1 Find the value of k for which f(x) = $\begin{cases} \frac{\sin 2x}{5x} & , x \neq 0 \\ k & , x = 0 \end{cases}$ is continuous at x = 0.

Q2 Find the relationship between a and b so that the function defined by

f(x) = $\begin{cases} ax+1 & ,\text{if } x \le 3 \\ bx+3 & ,\text{if } x > 3 \end{cases}$ is continuous at x = 3.

Q3 If f(x) = $\begin{cases} 3ax+b, & \text{if} \quad x>1 \\ 11 & \text{if} \quad x=1 \\ 5ax-2b, & \text{if} \quad x<1 \end{cases}$, continuous at x = 1,find the values of a and b.

Q4 Check whether the function f(x) = $x^2|x|$ is differentiable at x = 0 or not.

Q5 If y = $\sqrt{tan\sqrt{x}}$, prove that $\sqrt{x}\frac{dy}{dx} = \frac{1+y^4}{4y}$

Q6 If f(x) = |tan2x|, then find the value of f ′(x) at x = π/3.

Q7 If y = cosec($\cot^{-1}$(x)), then prove that $\sqrt{1+x^2}\frac{dy}{dx} - x = 0$.

Q8 If x = $e^{x/y}$, prove that $\frac{dy}{dx} = \frac{logx - 1}{(logx)^2}$

Q9 Check the differentiability of $f(x) = \begin{cases} x^2 + 1, 0 \le x < 1 \\ 3 - x, \quad 1 \le x \le 2 \end{cases}$ at x = 1.

Q10 If y = $\cos^3(\sec^2 2t)$, find dy/dt.

Q11 If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+logx)^2}$.

Q12 If xe^y = 1, then find the value of $\frac{dy}{dx}$ at x = 1.

Q13 Verify whether the function f defined by $\begin{cases} xsin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous or not.

Q14 Check for differentiability of the function *f* defined by $f(x)$= |x – 5|, at the point x = 5.

Q15 Differentiate $e^{x(1+logx)}$ w.r.t. x.

Answers: Q1 2/5 Q2 3a – 3b = 2b Q3 a = 3, b = 2 Q6 – 8

Q10 – 12$\cos^2(\sec^2 2t)\sin(\sec^2 2t)$ tan2t$\sec^2 2t$ Q12 – 1 Q15 $e^{x(1+logx)}(1 + x + xlogx)$

Short Questions

Q1 If $\sqrt{1+x^2} + \sqrt{1+y^2} = a(x-y), prove\ that\ \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

Q2 If y = $(tanx)^x$, then find $\frac{dy}{dx}$.

Q3 If x = $e^{\cos 3t}$ and y = $e^{\sin 3t}$, prove that $\frac{dy}{dx} = -\frac{ylogx}{xlogy}$

Q4 Show that $\frac{d}{dx}(|x|) = \frac{x}{|x|}, x \neq 0$

Q5 If xcos(p + y) + cospsin(p + y) = 0, prove that $cosp\frac{dy}{dx} = -cos^2(p+y), where\ p\ is\ a\ constant.$

Q6 If y = $(\log_e x)^x + x^{\log_e x}$ find $\frac{dy}{dx}$.

Q7 If x=a(cost + tsint) and y = a (sint – t cost) find $\frac{d^2y}{dx^2}$

Q8 If x = $a\sin^3\theta$, y = $b\cos^3\theta$, then find $\frac{d^2y}{dx^2}$ at θ = π/4.

Q9 *If* $y=\left(x+\sqrt{x^2+a^2}\right)^n$, *prove that* $\frac{dy}{dx}=\frac{ny}{\sqrt{x^2+a^2}}$

Q10 If $x\sqrt{1+y}+y\sqrt{1+x}=0$, for , $-1<x<1$, prove that $\frac{dy}{dx}=-\frac{1}{1+x^2}$

Answers:Q2 $(\tan x)^x(\tan x + x\sec^2 x)$ Q6 $(\log_e x)^x\left[\frac{1}{\log x}+\log(\log x)\right]+x^{\log_e x}\left[2\frac{\log x}{x}\right]$

Q7 $\frac{\sec^3 t}{at}$ Q8 $\frac{b}{3a^2}$ $4\sqrt{2}$

Chapter 6: Application of Derivatives

Q1. If the rate of change of volume of a sphere is equal to the rate of change of its radius, find the radius of the sphere.

Q2. An edge of a variable cube is increasing at the rate of 5 cm per second. How fast is the volume increasing when the side is 15 cm?

Q3. Find the rate of change of the area of a circle with respect to its radius "r" when r = 4cm.

Q4. The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2 cm/minute, when x= 10 cm and y = 6 cm, find the rates of change of the perimeter.

Q5. The radius of a balloon is increasing at the rate of 10 cm/sec. At what rate is the surface of the balloon increasing when the radius is 15 cm?

Q6. The length x of a rectangle is decreasing at the rate of 3cm/minute and the width y is increasing at the rate of 2cm/minute. When x=10cm and y= 6cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

Q7. For the function f(x) =$x^3 - 6x^2 - 1$, find the intervals:

(a) In which f (x) is increasing (b) in which f(x) is decreasing.

Q8. A particle moves along the curve 6y = x^3 + 2. Find the points on the curve at which the y- coordinate is changing 8 times as fast as the x- coordinate.

Q9. The surface area of a spherical bubble is increasing at the rate of 2 cm^2/s. Find the rate at which the volume of the bubble is increasing at the instant if its radius is 6 cm.

Q10. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the leader is 4 m away from the wall ?

Q11. Find the values of x for which y = $[x (x - 2)]^2$ is an increasing function.

Q12. Prove that : y= $\frac{4 \sin \theta}{2+ \cos \theta} - \theta$, is an increasing function in $\left[0, \frac{\pi}{2}\right]$.

Q13. Find the intervals in which the function f(x)= $x^3 - 12x^2 + 36x + 17$ is (a) increasing, (b) decreasing.

Q14. Find the intervals in which the function f given by

$$F(x) = x^3 + \frac{1}{x^3},\ x \neq 0 \text{ is}$$

(i) increasing (ii) decreasing

Q15. A square piece of tin of side 18 cm is to be made into a box without top by cutting a square from each corner and folding up the flaps to form a box. Find the maximum volume of the box.

Q16. A wire of length 28 cm is to be cut into two pieces. One of the two pieces is to be made into a square and the other into a circle. What should be the length of two pieces so that the combined area of them is minimum?

Q17. Show that the rectangle of maximum area that can be inscribed in a circle is a square.

Q18. Show that the right circular cylinder, open at the top, and of given surface area and maximum volume is such that its height is equal to the radius of the base.

Q19. Find the largest possible area of the right angled triangle whose hypotenuse is 5 cm.

Q20. An isosceles triangle of vertical angle 2θ is inscribed in a circle of radius a.

Show that the area of triangle is maximum when $\theta = \frac{\pi}{6}$.

Chapter 7: Integrals

SHORT ANSWER TYPE QUESTIONS

1. Find $\int (x^{\frac{3}{2}} + 2e^x - \frac{1}{x})dx$
2. Find $\int \frac{x^3-x^2+x-1}{(x-1)} dx$
3. $Find \int \frac{2-3sinx}{\cos^2 x} dx$
4. Find $\frac{\int \tan^4 \sqrt{x} \sec^2 \sqrt{x}}{\sqrt{x}} dx$
5. $Find \int \frac{1}{xlogx} dx$
6. $Find \int \frac{e^{2X}-e^{-2x}}{e^{2X}+e^{-2x}} dx$
7. $Find \int x\sqrt{(x+2)}dx$
8. $Find \int \frac{e^{\tan^{-1} x}}{1+x^2} dx$
9. $Find \int \frac{sinx}{(1+cosx)^2} dx$
10. $Find : \int cot\ x\ log\ sin\ x\ dx$
11. Find : $\int \frac{dx}{x^2-16}$
12. Find: $\int \frac{1}{(x+1)(x+2)} dx$
13. $Find: \int \frac{3x^2}{x^6+1} dx$
14. $Find \int \frac{x-1}{\sqrt{x^2-1}} dx$
15. $Find \int \frac{\sec^2 x}{\sqrt{tan^2 x+4}} dx$

Answer:

1. $\frac{2x^{\frac{5}{2}}}{5} + 2e^x - \log\lvert x\rvert + C$ 2. $\frac{x^3}{3} + x + c$ 3. 2tanx−3secx+C	8. $e^{\tan^{-1} x} + C$ 9. $\frac{1}{1+cosx} + C$ 10. $\frac{(\log(sinx))^2}{2} + C$ 11. $\frac{1}{8}\log\left\lvert\frac{x-4}{x+4}\right\rvert + C$

4. $\frac{2}{5}\tan^5\sqrt{x}+C$ 5. $loglogx+C$ 6. $\frac{1}{2}\log(e^{2x}+e^{-2x})+C$ 7. $\frac{2}{5}(x+2)^{\frac{5}{2}}-\frac{4}{3}(x+2)^{\frac{3}{2}}+C$	12. $\log\left\|\frac{x+1}{x+2}\right\|+C$ 13. $\tan^{-1}x^3$+C 14. $\sqrt{x^2-1}-\log\left\|x+\sqrt{x^2-1}\right\|+C$ 15. $\log\|tanx+\sqrt{\tan^2 x+4}+C$

LONG ANSWER TYPE QUESTIONS

1. $Find\int\frac{1}{1+tanx}dx$ 2. $Find\int xsin^{-1}xdx$ 3. $Find\int\frac{(x-3)e^x}{(x-1)^3}dx$ 4. $Find\int[\log(logx)+\frac{1}{(\log x)^2}]dx$
2. Find: $\int\sqrt{tanx}dx$ [NCERT EXEMPLAR]

Answer:

1. $\frac{x}{2}-\frac{1}{2}\log|cosx-sinx|+C$
2. $\frac{1}{4}(2x^2-1)\sin^{-1}x+\frac{x}{4}\sqrt{1-x^2}+C$
3. $\frac{e^x}{(x-1)^2}+C$
4. $xlog(logx)-\frac{x}{logx}+C$
5. $\frac{1}{\sqrt{2}}\tan^{-1}\frac{tanx-1}{\sqrt{2tanx}}+\frac{1}{2\sqrt{2}}\log|\frac{tanx-\sqrt{2tanx}+1}{tanx+\sqrt{2tanx}+1}|+C$

PREVIOUS YEARS ASKED QUESTION

1. $\int\sqrt{1+sin2x},\frac{\pi}{4}<x<\frac{\pi}{2}$ [CBSE, 2019]
2. $\int\frac{sec^2x}{\sqrt{4+tan^2x}}dx$ [CBSE, 2019]
3. $\int sinxsin2xsin3xdx$ [CBSE, 2019]
4. $\int\frac{1}{\sin(x-a)\cos(x-b)}dx$[CBSE, 2019]
5. $Find\int\frac{(x-5)e^x}{(x-3)^3}dx$[CBSE, 2019]

Chapter Test(20 marks)

1. If (d/dx) f(x) is g(x), then the antiderivative of g(x) is

   (a f(x) (b) f'(x) (c) g'(x) (d) None of these(1mark)

2. If ∫ 2^x dx = f(x) + C, then f(x) is

   (a)2^x (b)$2^x \log_e 2$ © $2^x / \log_e 2$ (d)2^{x+1}/x+1(1mark)

3. ∫$\cot^2x$ dx equals to

   (a)cot x – x + C (b) -cot x – x + C (c) cot x + x + C (d) -cot x + x + C(1mark)

4. If ∫ $\sec^2(7 – 4x)$dx = a tan (7 – 4x) + C, then value of a is

   (a) -4 (b) -¼ (c) 3 (d) 7(1mark)

Assertion and Reasoning

In the following question a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A)
(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A)
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

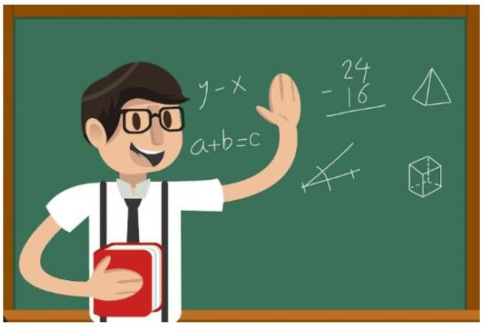
5. Assertion (A): If the derivative of the function x is $\frac{d}{dx}(x) = 1$, then its antiderivatives or integral is $\int 1dx = x + C$

Reason(R) : If $\frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n$, then the corresponding integral of the function is $\int x^n \ dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$

(1mark)

6. $Evaluate: \int \sqrt{1 - sin2x}dx. \frac{\pi}{4} < x < \frac{\pi}{2}$(2marks)
7. $Evaluate: \int \frac{dx}{x(1+logx)^2}$(2marks)
8. $Evaluate: \int \frac{dx}{9x^2+6x+10}$(2marks)
9. $\int \boldsymbol{e^x \left(\frac{1+sinx}{1+cosx}\right) dx}$(5marks)
10. Case Based Question

Mr. Dharmendra Kumar is a Maths teacher. One day he taught students that the Integral I = ∫ f (x) dx can be transformed into another form by changing the independent variable x to t by



substituting

Consider I = ∫ f (x) dx

$$Put\ x = g(t) so\ that\ \frac{dx}{dt} = g'(t). we\ write\ dx = g'(t)dt$$

$$I = \int f(x)dx = \int f\left(g(t)\right)g'(t)\ dt$$

This change of variable formula is one of the important tools available to us in the name of integration by substitution. Based on the above information, answer the following questions:

(e) $Evaluate: \int \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}\ dx$ (2marks)(ii) $Find\ \int \frac{1}{xlogx} dx$ (2marks)

Chapter Test (30 marks)

1. $\int e^x(cosx - sinx)dx\ is\ euqal\ to$

   (a) $e^x cosx + C$ (b) $e^x sinx + C$ (c) -$e^x cosx + C$(d) $-e^x sinx + C$(1mark)

2. $\int \frac{dx}{\cos^2 xsin^2x}$ is equal to

   (a) $\tan x + \cot x +$ C (b) $(\tan x + \cot x)^2 + C$ (c) $\tan x - \cot x + C$ (d) $(\tan x - \cot x)^2 + C$(1mark)

3. $\int \frac{(cos 2x - cos 2y)dx}{cosx - cosy}$ is equal to (1mark)

   $(a) 2(sinx + xcosy) + C$ (b) $2(sinx - xcosy) + C$ (C) $2(sinx + 2xcosy) + C$ (d) $2(sinx - 2xcosy) + C$

4. $\int \tan^{-1}\sqrt{x}\, dx\ is\ equal\ to$ (1mark)

   $(a)(x+1)\tan^{-1}\sqrt{x} - \sqrt{x} + C$ (b) $x\tan^{-1}\sqrt{x} - \sqrt{x} + C(c)-x\ \tan^{-1}\sqrt{x} + \sqrt{x} + C(d)-(x+1)\tan^{-1}\sqrt{x} + \sqrt{x} + C$

5. $\int \frac{e^x(1-x)^2}{(1+x^2)^2} dx\ is\ euqal\ to$(1mark)

$(a)\frac{e^x}{1+x^2}+C$ (b)$-\frac{e^x}{1+x^2}+C$ (c)$\frac{e^x}{(1+x^2)^2}+C$(d) $-\frac{e^x}{(1+x^2)^2}+C$

6. $\int \frac{x^9}{(4x^2+1)^6} dx\ is\ equal\ to$ (1mark)

$(a)\frac{1}{5x}\left(4+\frac{1}{x^2}\right)^{-5}+C$ (b)$\frac{1}{5}\left(4+\frac{1}{x^2}\right)^{-5}+C(c)\frac{1}{10x}(4+1)^{-5}+C$ (d) $\frac{1}{10}\left(4+\frac{1}{x^2}\right)^{-5}+C$

7. $\int \frac{dx}{(x+2)(x^2+1)} = alog|1+x^2| + btan^{-1}x + \frac{1}{5}\log|x+2| + C, then$

(a) $a=-\frac{1}{10}, b=-\frac{2}{5}$ (b) $a=\frac{1}{10}, b=-\frac{2}{5}$ (c) $a=-\frac{1}{10}, b=\frac{2}{5}$ (d) $a=\frac{1}{10}, b=\frac{2}{5}$(1mark)

8.

In the following question a statement of assertion (A) is followed by a statement of reason (R).

Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A)

(b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

$$\text{Assertion}(A): \int \frac{1}{\sqrt{x^2+2x+10}} dx = \sin^{-1}\left(\frac{x+1}{3}\right)+C$$

Reason(R): $\text{If a} > 0, \text{b}^2-4ac<0\ then \int \frac{1}{\sqrt{ax^2+bx+c}} dx = \frac{1}{\sqrt{a}}\sin^{-1}\left(\frac{2ax+b}{\sqrt{b^2-4ac}}\right)$(1mark)

9. $Find: \int \frac{(e^{6logx}-e^{5logx})}{(e^{4logx}-e^{3logx})}\ dx$ (2marks)

10. $Find: \int \tan^2 x \sec^4 x\, dx$(2marks)

11. $Find: \int \frac{dx}{1+cosx}$(2marks)

12. $Find\ \int e^{\tan^{-1}x} \frac{1}{1+x^2} dx$ (2marks)

13. $\int \frac{2x-1}{(x-1)(x+2)(x-3)} dx$(5marks)

14. $\int \frac{x^2}{(x^2+a^2)(x^2+b^2)} dx$(5marks)

DEFINITE INTEGRAL

MCQ(1MARK QUESTIONS)

1.$\int_0^{\pi/2} \frac{sinx - COSX}{1+sinxcosx}$dx is equal to

A) π B)0 C) $\int_0^{\pi/2} \frac{2sinx}{1+sinxcosx}$ D) $\frac{\pi^2}{4}$

2. Evaluate $\int_0^{2\pi} (cos^5 x)\, dx$

A) 0 B) 4 C) 6 D) 7

3.$\int_{-\pi}^{\pi} x(cosx)\, dx$ = ...

A) 1 B) -1 C) 0 D) π/2

4. $\int_{-\pi}^{\pi} x(sinx)\, dx$ = ...

A) π B) 2π C) 0 D) π/2

ASSERTION-REASON BASED QUESTIONS

In the following questions, a statement of assertion (A) is followed by a statement ofReason (R). Choose the correct answer out of the following choices.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true but R is not the correct explanation of A.

(c) A is true but R is false.

(D)A is false but R is true.

Assertion: $\int_{-3}^{3} (x^3 + 5)\quad dx$ = 30

Reason: f(x) = x^3 + 5 is an even function

Explanation: $\int_{-3}^{3} (x^3 + 5)\quad dx$ =30 is true. But f(x) is neither even nor odd. Hence R is false.

Hence A is true.

SHORT ANSWER

1.Evalate: $\int_1^4 |x-1| + |x-2| + |x-3| dx$

2.Evaluate: $\int_{\frac{\pi}{2}}^{\pi} \sqrt{1+\cos 2x}\, dx$ (JEE MAIN 2019)

3.Evaluate: $\int_0^2 [x^2] dx$ (JEE ADVANCE 2018)

Long Answer

1Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}} dx$

Q2 Evaluate: $\int_0^{\pi} \log(1 + cosx)\, dx$

PRACTICE QUESTIONS

1. Evaluate $\int_{-2}^{2}(x^3 + 1)dx$

A) 0 B) 4 C) 6 D) 7

2. Evaluate $\int_0^{2\pi}(cos^5 x)\, dx$

A) 0 B) 4 C) 6 D) 7

3. Evaluate $\int_2^{3}(3^x . log\ 3)dx$

A) 0 B) 4 C) 18 D) 15

4. Evaluate $\int_0^{1}\left(e^{x^2} . x\right)dx$

A) 0 B) $\frac{1}{2}(e + 1)$ C) $\frac{1}{2}(e - 1)$ D) $\frac{1}{2}(e^2 - 1)$

5. Evaluate secx. (sec x + tan x) dx

A) tan x + sec x + C B) tan x - sec x + C

C) tan^2 x + sec x + C D) tan^2 x - sec x + C

Answer MCQ: 1.B, 2.A, 3.C, 4.C , 5.A

SHORT ANSWER TYPE QUESTIONS

1. Evaluate $\int_0^{2\pi} |\cos x|\, dx$
2. Evaluate: $\int_0^{1}(3x^2 + 2x + k)\ \ dx = 0, find\ the\ value\ of\ k.$
3. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{4+5\cos\ x}$
4. Evaluate: $\int_0^{\frac{\pi}{2}} \log(1 + tan\theta)\, d\theta$
5. Evaluate: $\int_0^{\frac{\pi}{2}} \frac{(x+sinx\ )}{1+cosx} dx$
6. Evaluate: $\int_0^{\frac{\pi}{2}} \log(\sin x)\, dx$
7. Evaluate: $\int_0^{\pi/2}\left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx$
8. Evaluate: $\int_0^{\pi} \frac{x}{a^2 cos^2 x + b^2\ sin^2 x} dx$
9. $Evaluate$: $\int_{-1}^{2} |x^3 - x| dx$

10. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\tan x}}{1+\sqrt{\tan x}}dx$

11. Evaluate: $\int_0^3 [x]dx$

12. Evaluate $\int_{-2}^{2} \frac{x^2}{1+5^x}\, dx$

13. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{1+\sqrt{\cot x}}dx$

14. *Evaluate*: $\int_0^{2\pi} \cos^4 x dx$

15. Evaluate: $\int_0^{\frac{\pi}{4}} \ln(1+\tan x)\, dx$

HOTS

1. Evaluate $\int_1^4 [|x-1|+|x-2|+|x-3|]dx$

2. Evaluate $\int_0^1 tan^{-1}(1-x+x^2)dx$

3. Evaluate $\int_{-\pi}^{\pi} \frac{cos^2 x}{1+a^x}\, dx,\ a>0.$

4. Evaluate $\int_{-\frac{1}{2}}^{\frac{1}{2}} \left([x]+log\left(\frac{1+x}{1-x}\right)\right) dx.$

5. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^{100} x}{\sin^{100} x+\cos^{100} x}dx$ (Ans. $\frac{\pi}{2}$) JEE Advance 2018

6. Evaluate: $\int_0^3 |x^2-3x+2|dx$ (Ans. 22) JEE MAIN 2024

7. Evaluate: $\int_{\frac{\pi}{2}}^{\pi} \sqrt{1+\cos 2x}dx$ (JEE MAIN 2019) (Ans $\sqrt{2}$

8. Evaluate: $\int_0^2 [x^2]dx$ (JEE ADVANCE 2018) (Ans. 6 - $\sqrt{2}$ +5 $\sqrt{3}$)

Chapter 8: APPLICATIONS OF THE INTEGRALS

MCQ

Q1. Find the area enclosed by curve $4x^2 + 9y^2 = 36$

(a) 6л sq units (b) 4л sq units (c) 9л sq units (d) 36л sq units

Q2. The area of the region bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

a. ⅜ sq. units (b) ⅝ sq. units (c)⅞ sq. units (d) 9/8 sq. units

Answer: (d) 9/8 sq. units

Q3. The area enclosed between the graph of $y = x^3$ and the lines x = 0, y = 1, y = 8 is

(a) 7 (b) 14 (c) 45/4 (d)None of these

Q4. The area of the region bounded by the curve $y^2 = x$, the y-axis and between y = 2 and y = 4 is

(a) 52/3 sq. units (b)54/3 sq. units (c)56/3 sq. units (d)None of these

Q5. Area of region bounded by the curve $y^2 = 4x$,and its latus rectum above x axis

(a) 0 sq units (b) 4/3 sq units (c) 3/3 sq units (d) 2/3 sq units

Ans (b) 4/3 sq units

Q6. Area of region bounded by $y= x^3$, x axis , x=1 and x=-2

a)-9 sq units (b) -15/4 sq units (c) 15/4 sq units (d) 17/4 sq units

Ans (b) -15 /4

Q7. Area of region bounded by curve y=x and $y = x^3$ is

(a) 1/2 sq units (b) 1 /4 sq units (c) 9/2 sq units (d) 9/4 sq units

Ans. (a) 1/2 sq units

Q8. The area enclosed by the circle $x^2+y^2 = 2$ is equal to:

(a) 4л sq units (b) 2√2л sq units (c) $4л^2$ sq units(d) 2л sq units

Ans. (d) 2л sq units

Q9. The area of the region bounded by the parabola $y = x^2$ and $y = |x|$ is

(a) 3 (b) 1 / 2 (c) 1/3 (d) 2

Ans. (c) 1/3

Q10. The area of the region enclosed by the parabola $x^2 = y$, the line y = x + 2 and the x-axis, is

(a) 5 / 9 (b) 9 / 5 (c) 5 /6 (d) 2 /3

Ans (c) 5 /6

ASSERTION - REASON TYPE QUESTIONS:

Directions: Each of these questions contains two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

(a) Assertion is correct, Reason is correct; Reason is a correct explanation for assertion.

(b) Assertion is correct, Reason is correct; Reason is not a correct explanation for Assertion

(c) Assertion is correct, Reason is incorrect

(d) Assertion is incorrect, Reason is correct

Q.11 Assertion : The area bounded by the curve y = cos x in I quadrant with the coordinate axes is 1 sq. unit. Reason : $\int_0^{\frac{л}{2}} \cos x \, dx = 1$

Ans (a) $\int_0^{\frac{л}{2}} \mathbf{\cos x \, dx} = [\mathbf{\sin x}]_0^{\pi/2}$ = $\sin\frac{л}{2}$ - 0 = 1

Q.12 Assertion : The area bounded by the curves $y^2 = 4a^2 (x – 1)$ and lines x = 1 and y = 4a is 16a 3 sq. units.

Reason : The area enclosed between the parabola $y^2 = x^2 – x + 2$ and the line y = x + 2 is 8 3 sq. units

Ans. (c)

Q.13Assertion : The area bounded by the circle $x^2 + y^2 = a^2$ in the first quadrant is given by

$$\int_0^a \sqrt{a^2 - x^2} \, dx$$

Reason : The same area can also be found by $\int_0^a \sqrt{a^2 - y^2} \, dy$

Ans. (b)

Q.14Assertion : The area bounded by the circle y = sin x and y = –sin x from 0 to л is 3 sq. unit.

Reason :The area bounded by the curves is symmetric about x-axis.

Ans. (d)

Short Answer Type Question

Q. Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$

Q. Find the area bounded by the line y = x, x-axis and lines x = – 1 to x = 2.

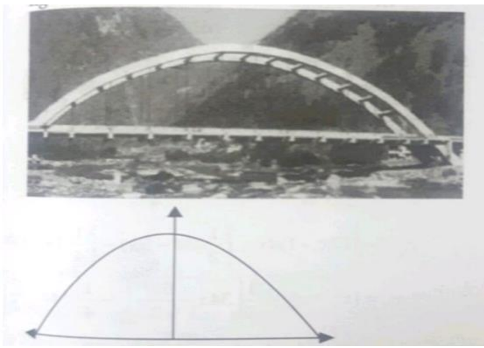
Q. Find the area between the curves y = x and $y = x^3$.

Long Answer Type Questions :

Q. Find the area of the region enclosed by the parabola $x^2 = y$, the line $y = x + 2$ and the x-axis,

CASE STUDY QUESTION : 1

The bridge connects two hills 100 feet apart. The arch of the bridge is in a parabolic form.Based on the information given above, answer the following questions:



(i) the equation of the parabola designed on the bridge is

(a) $x^2 = 250y$

(b) $x^2 = -250y$

(c) $y^2 = 250x$

(d) $y^2 = -250x$

(ii) the value of the integral

(a) 1000 / 3 (b) 2500 / 3 (c) 1200 (d) 0

(iii) the integrand of the integral function.

(a) Even (b) Odd (c) Neither odd nor even (d) None

(iv) The area formed by the curve x 2 =250 y , x axis , y=0 and y=10 is

(a) 1000√2 / 3 (b) 4 / 3 (c) 2000 / 3 (d) 0

(v) The area formed by the curve x 2 =250y, y axis , y=2 and y=4 is

(a) 1000√2 / 3 (b) 0 (c) 1000 / 3 (d) none of these

Answers (i)b (ii) a (iii) a (iv) c (v) d

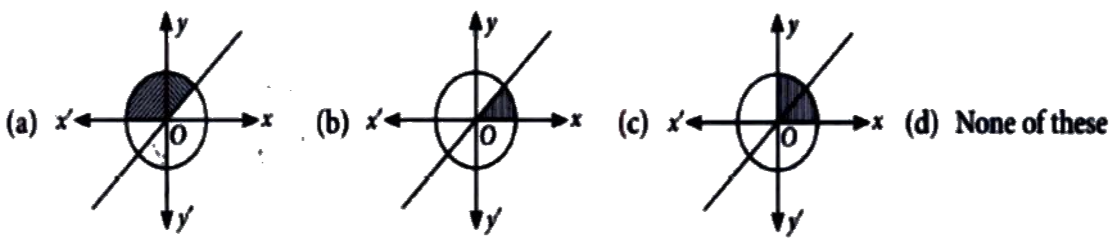
CASE STUDY QUESTION : 2

Consider the curve and line y = x in the first quadrant based on the given information, answer the following questions .

(i) Paint the intersection of both the given curve is

(a) (0,4) (b) (0, 2√2) (c) (2√2, 2√2) (d) (2√2,4)

(ii) Which of the following shaded portion represent the area bounded by the given two curves ?



(iii) The value of the integral $\int_0^{2\sqrt{2}} x\,dx$ is

(a) 0 (b) 1 (c) 2 (d) 4

(iv) The value of the integral $\int_0^{2\sqrt{2}} \sqrt{16 - x^2}dx$ is

(i) 2(л-2) (b) 2(л-8) (c) 4(л-2) (d) 4(л+2)

(v) The area bounded the given curves is

(i) 3л sq. units (ii) л/2 sq. units (iii) л sq. units (d) 2л sq. units

<u>CASE STUDY QUESTION : 3</u>

Consider the following equations of curves y = cosx, y = x+1 and y=0. On the basis ofabove information, answer the following questions.

(i) The curves y = cosx and y =x+1 meet at(a) (1, 0) (b) (0 ,1) (c) (1 ,1) (d) (0, 0)

(ii) y = cosx meet x-axis at(a) (-л/2, 0) (b) (л/2, 0) (c) both (a) and (b) (d) None of these.

(iii) Value of the integral $\int_{-1}^{0}(x + 1)$dx is(a) 1 /2 (b) 2 /3 (c) 3 /4 (d) 1 /3

(iv) Value of the integral $\int_0^{\pi/2} \cos x \, dx$ is (a) 0 (b) -1 (c) 2 (d) 1

(v) Area bounded by the given curves is

(i) 1 /2 sq. units (ii) 3 /2 sq. units (iii) 3 /4 sq. units (d) 1 /4 sq. units

<u>Short Answer type questions (Unsolved)</u>

Q1. Find the area enclosed between curves y = x^2 + 2, y = x, x = 0, x = 3

Ans. 21 /2 sq. units

Q2. Find the area of the region bounded by the curve y=sinx between the lines x=0 , x=π/2 and the x-axis.

Ans 4 sq units

Q3. Find the area enclosed between curves y = 4x – x^2 , 0 ≤ x ≤ 4, x-axis

Ans. 32 /3 sq. units

Q4. Find the area enclosed between curves y = x^2 + 2, y = x, x = 0, x = 3

Ans. 21 /2 sq. units

Q5. Find the area $\{(x, y): x^2 + y^2 \le 1 \le x + y\}$ { (x , y) : $x^2 + y^2 < 1 <$ x+y}

Ans. π/4 - 1/ 2 sq. units.

Q6. Find the area enclosed between y^2 = 4ax and its latus rectum.

Ans. 8 a^2/3 sq. units.

Q7. Find the area enclosed between curves y = x^3, x = - 2, x = 1, y = 0

Ans. 15 /4 sq. units

Q8. Find the area bounded by the curve y = cos x between x = 0 and x = 2π

Ans. 4 sq. units

Long Answer Type Questions : (Unsolved)

Q1. Find the area of the region bounded by the curve $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Ans. πab.

Q2. Using the method of integration find the area bounded by the curve $|x| + |y| = 1$

Ans. 4 sq. units

Q3. Find the area lying above x-axis and included between the circle $x^2 + y^2 = 8x$ and the parabola $y^2 = 4x$

.

Ans. 4π + 32/3 sq. units

Q4. Draw the rough sketch and find the area of the region bounded by two parabolas $4y^2 = 9x$ and $3x^2 = 16y$ by using method of integration.

Ans. 4 sq. units

Q5. Find the area of the region bounded by the line y = 3x + 2, the x-axis and the ordinates x = –1 and x = 1.

Ans. 13 /3 sq. units.

CHAPTER 9: DIFFERENTIAL EQUATIONS

Q.1	Find the order and degree of the differential equation $x^2\frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^4$
Q.2	Find the order and degree of the differential equation $\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 = 2\ x^2\log\left(\frac{d^2y}{dx^2}\right)$
Q.3	Find the order and degree of the differential equation $y = e^{\sin\left(\frac{d^3y}{dx^3}\right)^2} + \left(\frac{dy}{dx}\right)^4$
Q.4	Find the sum of order and degree of the following differential equation $\frac{d^2y}{dx^2} + \sqrt[3]{\frac{dy}{dx}} + (1+x) = 0$
Q.5	Find the solution of the differential equation $\frac{dy}{dx} = x^3e^{-2y}$
Q.6	Write the integrating factor of the differential equation $\sqrt{x}\frac{dy}{dx} + y = e^{-2\sqrt{x}}$
Q.7	Write the integrating factor of the differential equation $(1+y^2) + (2xy - \cot y)\frac{dy}{dx} = 0$
Q.8	Find the sum of order and degree of the following differential equation $\frac{d}{dx}\left\{\left(\frac{d}{dx}\right)^3\right\} = 0$
Q.9	Find the degree of the following differential equation $\frac{d^2y}{dx^2} + \sqrt{1+\left(\frac{dy}{dx}\right)^2} = 0$
Q.10	Find the order and degree of the following differential equation $\sqrt[3]{\frac{d^2y}{dx^2}} = \sqrt{\frac{dy}{dx}}$
Q.11	Find the product of the order and degree of the following differential equation: $x\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 + y^2 = 0$
Q.12	Find The integrating factor of the differential equation $\frac{dy}{dx}(x\log x) + y = 2\log x$
Q.13	Solve: $\frac{dy}{dx} + y = e^{e^x}$

Q.14	Find the order and degree of the differential equations $$\left(1-\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$$
Q.15	for what value of n is the $\frac{dy}{dx} = \frac{x^3-y^n}{x^2y+xy^2}$ a homogeneous differential equation:
Q.16	If p is the order of the differential equation and q is the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^3 + 2y = 0$, then find the value of $3p - q$.
Q.17	The number of arbitrary constants in the particular solution of differential equation of order 4
Q.18	How many arbitrary constants in the general solution of differential equation of order 3.
Q.19	Find the sum of the degree and the order for the following differential equation : $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)^4 = 0$
Q.20	Find the order of the differential equation of all circles of radius *r*, having centre on *y*-axis and passing through the origin .
	ASSERTION- REASON TYPE QUESTIONS Directions : Im the following questions a statement of assertion (A) is followed by a statement of reason(R).Choose the correct answer out of the following choices. (a) : Both A and R are true and R is the correct explanation of A. (b) : Both A and R are true but R is NOT the correct explanation of A. (c): A is true but R is false. (d): A is false but R is true.
Q.21	Assertion (A): The order and degree of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \frac{dy}{dx} + 5$ are 2 and 1 respectively. Reason (R): Order of differential equation is highest power of $\frac{dy}{dx}$ which is 2 in given differential equation after making it free from radical and degree is also 2 as highest number of differentiation done in the equation is two.
Q.22	Assertion (A): The solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$ with initial condition x=1 and y= 1 is x=y. Reason (R): Separation of variable method can be used to solve the differential equation.

Q.23	Assertion(A): The order of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^2+\cos\left(\frac{dy}{dx}\right)\right]^{\frac{3}{4}}=\frac{d^3y}{dx^3}$ is 3. Reason (R): The degree of differential equation $\left[1+\left(\frac{dy}{dx}\right)^2+\cos\left(\frac{dy}{dx}\right)\right]^{\frac{3}{4}}=\frac{d^3y}{dx^3}$ is not defined.
Q.24.	**1.** Assertion (A): Solution of the differential equation $(1+x^2)\frac{dy}{dx}+y=\tan^{-1}x$ is $ye^{\tan^{-1}x}=(\tan^{-1}x-1)e^{\tan^{-1}x}+C$ Reason (R) : The differential equation of the form $\frac{dy}{dx}+Py=Q$, where P, Q be the functions of x or constant, is a linear type differential equation.
Q.25	**2.** Assertion (A): Solution of the differential equation $e^{dy/dx}=x^2$ is y = 2(*x*log*x* – *x*) + C. Reason (R): The differential equation $\frac{d^2y}{dx^2}+y=0$ has degree 1 and order 2

ANSWERS

1. order2, degree 1	2. order2, degree N.D.	3. order3,degree N.D.	4. 5	5. $1/2(e^{2y})=x^4+c$
6. $e^{2\sqrt{x}}$	7. $1+y^2$	8. 3	9. 2	10. order 2, degree 2
11. 4	12. logx	13. $ye^x=e^{e^x}+c$	14. order 2,degree 2	15. 3
16. 4	17. 0	18. 3	19. order 3, degree 1	20. 1
21. a	22. a	23. b	24. a	25. b

2 MARKS QUESTIONS

Q.1	Solve the differential equation $ydx - x\,dy = 0$
Q.2	Find particular solution of the differential equation: $2y.e^{\frac{x}{y}}\,dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$ given that $x=0$ when $y=1$.
Q.3	Verify the given functions is the solution of the corresponding differential equation; y = $x^2+2x+c : y'-2x-2=0$. $y=\cos x+C \;:\; y'+\sin x=0$.
Q.4	Solve the differential equation $(1+x^2)\frac{dy}{dx}+y=tan^{-1}x$
Q.5	Find particular solution $x(x^2-1)\frac{dy}{dx}=1, y=0$ when x = 2

Q.6	Solve: $\frac{dy}{dx} + y = \cos x - \sin x.$
Q.7	Solve: $(x + y)\frac{dy}{dx} = 1.$
Q.8	The solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is
Q.9	Solve: $\frac{dy}{dx} - 3y\cot x = \sin 2x; y = 2\ when\ x = \frac{\pi}{2}$
Q.10	Solve the differential equation xdy- ydx =$\sqrt{x^2 + y^2}$ dx
Q.11	Solve :$secx\frac{dy}{dx} = y + sinx$
Q.12	Solve : $(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$
Q.13	Solve differential equation $3e^x \tan y dx + (1-e^x)\sec^2 y dy=0$ given that y= $\frac{\pi}{4}$, when x= 1.
Q.14	Solve the differential equation: $\frac{dy}{dx} = 1 + x + y + xy.$
Q.15	Solve $\frac{dy}{dx} = e^{-y}\cos x$
Q.16	Find the solution of differential equation $3e^x \tan y dx + (1-e^x)\sec^2 y dy = 0$.
Q.17	Solution of the equation $(1-x^2)dy + xydx = xy^2 dx$
Q.18	Solution of differential equation $2xy\frac{dy}{dx} = x^2 + 3y^2$ is
Q.19	Solve the following differential equation: $\frac{dy}{dx} = x^3 \operatorname{cosec} y$, given that y(0) = 0.
Q.20	Solve the differential equation: $x\frac{dy}{dx} - y = x^2$
Q.14	$\frac{dy}{dx} = 1 + x + y + xy$ $\Rightarrow \frac{dy}{dx} = (1+x)(1+y) \Rightarrow \frac{dy}{1+y} = (1+x)dx$ On integrating, we get $\log(1+y) = \frac{x^2}{2} + x + c$.

03 MARKS QUESTIONS

Q.1	Solve: $(x^2 - y^2)\,dx + 2xy\,dy = 0$ given that y=1 when x=1.
Q.2	Find the particular solution of differential equation : $(\tan^{-1} y - x)dy = (1 + y^2)dx$ Given when x =0 , y =0 .
Q.3	Solve: $\frac{dy}{dx} = \frac{(2y-x)}{(2y+x)}$ $if\ y = 1\ when\ x = 1.$
Q.4	Solve the differential equation: $ye^{\frac{x}{y}}dx = (xe^{\frac{x}{y}} + y^2)dy$
Q.5	Find the particular solution of the differential equation: (x + y) dy + (x – y) dx = 0 given that, when x = 1, y =1.
Q.6	Find particular solution $x(x^2 - 1)\frac{dy}{dx} = 1, y = 0$ when x = 2 .
Q.7	Find the equation of a curve passing through (2, 1) if the slope of the tangent to the curve at any point (x, y) is $(x^2 + y^2)/2xy$.
Q.8	Find The solution of the differential equation $(3xy + y^2)dx + (x^2 + xy)dy = 0$
Q.9	The solution of the equation $\frac{dy}{dx} = \frac{y}{x}\left(\log\frac{y}{x} + 1\right)$ is
Q.10	Find the general solution of the differential equation $(2x - y + 1)dx + (2y - x + 1)dy = 0$ is
Q.11	The solution of the equation $(x + 2y^3)\frac{dy}{dx} - y = 0$ is
Q.12	The general solution of the differential equation $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$
Q.13	The solution of the differential equation $\frac{dy}{dx} = \frac{x - y + 3}{2(x-y)+5}$ is
Q.14	Given that $dy/dx = e^{-2y}$ and y = 0 when x = 5. Find the value of x when y = 3.
Q.15	Find the equation of the curve through the point (1,0) and whose slope is $\frac{y-1}{x^2 + x}$
Q.16	Solve the differential equation $(x^2 - 1)\frac{dy}{dx} + 2xy = \frac{2}{x^2-1}$
Q.17	$xy\frac{dy}{dx} = (x+2)(y+2)$, find the equation of the curve passing through the point (1,-1)
Q.18	Solve the differential equation $x\frac{dy}{dx} = y - x\tan\frac{y}{x}$
Q.19	If The rate of increase of bacteria in a certain culture is proportional to the number present. If it double in 5 hours then in 25 hours, how many bactria will be there?
Q.20	In culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number

	present?
Q.21	At any point (x, y) of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point (– 4, –3). Find the equation of the curve given that it passes through (–2, 1).
Q.22	In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years (e0.5= 1.648).

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Chapter-10 Vector Algebra

MULTIPLE CHOICE QUESTIONS

Q1. If $\vec{a}$ and $\vec{b}$ are the vectors forming consecutive sides of a regular hexagon *ABCDEF*, then the vector representing the side *CD* is:

(a) $\vec{a}+\vec{b}$ (b) $\vec{a}-\vec{b}$ (c) $\vec{b}-\vec{a}$ (d) $-(\vec{a}+\vec{b})$

Q2. The vector cosα cosβ $\hat{\imath}$ + cosα sinβ $\hat{\jmath}$ + sin α $\hat{k}$ is a :

(a) null vector (b) unit vector (c) constant vector (d) none of these

Q3. The vector in the direction of the vector $\vec{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ that has a magnitude 9 is:

(a) $\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ (b) $\frac{1}{3}(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ (c) $3(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$ (d) $9(\hat{\imath} - 2\hat{\jmath} + 2\hat{k})$

Q4. The projection of the vector $\vec{a} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$ on $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ is:

(a) $\frac{\sqrt{14}}{2}$ (b) $\frac{14}{\sqrt{2}}$ (c) $\sqrt{14}$ (d) 7

Q5. The p.v.'s of the points *A, B, C* are $2\hat{\imath} + \hat{\jmath} - \hat{k}, 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\hat{\imath} + 4\hat{\jmath} - 3\hat{k}$ respectively. These points:

(a) form an isosceles triangle (b) form a right triangle

(c) are collinear (d) form a scalene triangle

Q6. The p.v.'s of the points *A, B, C* are $\hat{\imath} + x\hat{\jmath} + 3\hat{k}, 3\hat{\imath} + 4\hat{\jmath} + 7\hat{k}$ and $y\hat{\imath} - 2\hat{\jmath} - 5\hat{k}$ respectively are collinear, then (x, y) = ?

(a) (2, -3) (b) (-2, 3) (c) (-2, -3) (d) (2, 3)

Q7. If $\vec{a}$ and $\vec{b}$ be two vectors such that $\lfloor\vec{a}\rfloor = |\vec{b}| = \sqrt{2}$ and $\vec{a}.\vec{b}$ =−1, then the angle between $\vec{a}$ and $\vec{b}$ is:

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{2\pi}{3}$ (d) none of these

Q8. If $\vec{a} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, then angle between $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{2}$ (d) $\frac{2\pi}{3}$

Q9. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then:

(a) $\lfloor\vec{a}\rfloor = |\vec{b}|$ (b) $\vec{a} \perp \vec{b}$ (c) $\vec{a} \parallel \vec{b}$ (d) none of these

Q10. If $\vec{a}, \vec{b}$ and $\vec{c}$ are mutually perpendicular unit vectors, then value of $|\vec{a} + \vec{b} + \vec{c}|$ is:

(a) 1 (b) $\sqrt{2}$ (c) $\sqrt{3}$ (d) 2

In the following questions 11 and 12, a statement of assertion (A) is followed by a statement of reason (R).

Mark the correct choice as:

(a) Both Assertion (A) and Reason (R) are true and Reason(R) is the correct explanation of assertion (A). (b) Both Assertion (A) and Reason (R) are true but Reason(R) is not the correct explanation of assertion (A). (c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

11. Assertion (A): The value of $\hat{\imath}.\left(\hat{\jmath} \times \hat{k}\right) + \hat{\jmath}.\left(\hat{\imath} \times \hat{k}\right) + \hat{k}.\left(\hat{\imath} \times \hat{\jmath}\right)$ is 1.

Reason (R): Since, $\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath}. = \hat{k}.\hat{k} = 0$

12. Assertion (A): The direction of cosines of vector $\vec{a} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ are $\frac{2}{\sqrt{45}}, \frac{4}{\sqrt{45}}, \frac{5}{\sqrt{45}}$

Reason (R): A vector having zero magnitude and arbitrary direction is called 'zero vector' or 'null vector'

ANSWERS

1 c, 2 b, 3 b , 4 a, 5 a, 6 a, 7 c , 8 c, 9 b, 10 c, 11 c, 12 b

SECTION – B Questions 13 to 16 carry 2 marks each

13. Given $\vec{p} = 3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$, $\vec{a} = \hat{\imath} + \hat{\jmath}$, $\vec{b} = \hat{\jmath} + \hat{k}$, $\vec{c} = \hat{\imath} + \hat{k}$ and $\vec{p} = x\hat{a} + y\hat{b} + z\hat{c}$, then find the value of x, y, z.

14. Using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5).

15. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 18$.

16. If $\vec{a}, \vec{b}, \vec{c}$ are three mutually perpendicular vectors of equal magnitudes, prove that $\vec{a} + \vec{b} + \vec{c}$ is equally inclined with the vectors $\vec{a}, \vec{b}, \vec{c}$.

ANSWERS

13 x = 1/2 , y = 3/2, z = 5/2. 14 $\sqrt{61}$ /2 .15 $\vec{d} = 64\hat{\imath} - 2\hat{\jmath} - 28\hat{k}$

SECTION – C Questions 17 to 19 carry 3 marks each

17. Show that the points A(1, 2, 7), B(2, 6, 3) and C(3, 10, –1) are collinear.

18. Find a unit vector perpendicular to each of the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$,

$\vec{b} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$.

19. The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to its diagonal. Also, find its area.

ANSWERS

18 $\frac{-1}{\sqrt{6}}\hat{\imath} + \frac{2}{\sqrt{6}}\hat{\jmath} - \frac{2}{\sqrt{6}}\hat{k}$, 19 $\frac{3}{7}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{2}{7}\hat{k}$, $11\sqrt{5}$sq. unit.

SECTION – D Questions 20 to 22 carry 5 marks.

20. The magnitude of the vector product of the vector $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of vectors $2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\lambda\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ is equal to $\sqrt{2}$. Find the value of λ.

21. If $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ and $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$ show $(\vec{a} - \vec{d})$ is parallel to $\vec{b} - \vec{c}$,where $\vec{a} \neq \vec{d}$ and $\vec{b} \neq \vec{c}$.

22. If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$,and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

ANSWERS

20 $\lambda = 1$, 22 $\vec{\beta}_1 = \frac{1}{2}(3\hat{\imath} - \hat{\jmath})$, $\vec{\beta}_2 = \frac{1}{2}\hat{\imath} + \frac{3}{2}\hat{\jmath} - 3\hat{k}$

SECTION – E (Case Study Based Questions) Questions 23 & 24 carry 4 marks each.

23. Case-Study 1: Read the following passage and answer the questions given below.
Solar panels have to be installed carefully so that the tilt of the roof, and the direction to the sun, produce the largest possible electrical power in the solar panels. A surveyor uses his instrument to determine the coordinates of the four corners of a roof where solar panels are to be mounted. In the picture, suppose the points are labelled counter clockwise from the roof corner nearest to the camera in units of meters P1 (6, 8, 4), P2 (21, 8, 4), P3 (21, 16, 10) and P4 (6,16,10).

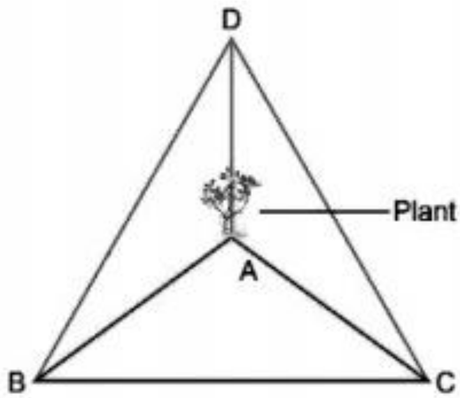
(i) Find the components to the two edge vectors defined by $\vec{A}$ = PV of P2 – PV of P1 and $\vec{B}$ = PV of P4 – PV of P1 where PV stands for position vector.

(ii) (a) Find the magnitudes of the vectors $\vec{A}$ and $\vec{B}$.

(b) Find the components to the vector $\vec{N}$, perpendicular to $\vec{A}$ and $\vec{B}$ and the surface of the roof.

24. Case-Study 2: Read the following passage and answer the questions given below.

Raghav purchased an air plant holder which is in shape of tetrahedron. Let A, B, C, D be the coordinates of the air plant holder where A = (1, 2, 3), B= (3, 2, 1), C = (2, 1, 2), D = (3, 4, 3).



(i) Find the vector $\overrightarrow{AB}$. (1)

(ii) Find the vector $\overrightarrow{CD}$. (1)

(iii) Find the unit vector along $\overrightarrow{BC}$ vector. (2)

OR

(iii) Find the area (ΔBCD). (2)

ANSWERS

23 (i) Components of $\vec{A}$ are 15, 0, 0, Components of $\vec{B}$ are 0, 8, 6

(ii) (a) $\left|\vec{A}\right|$ = 15 unit, $\left|\vec{B}\right|$ = 10 unit

(b) Components of $\vec{N}$ are 0, -90, 120

24 (i) $\overrightarrow{AB} = 2\hat{i} - 2\hat{k}$ (ii) $\overrightarrow{CD} = \hat{i} + 3\hat{j} + \hat{k}$

(iii) unit vector along $\overrightarrow{BC}$ vector = $\frac{-1}{\sqrt{3}}\hat{\imath} - \frac{1}{\sqrt{3}}\hat{\jmath} + \frac{1}{\sqrt{3}}\hat{k}$

OR

(ii) area (ΔBCD) = $\sqrt{6}$ sq. units

CHAPTER-11 : THREE DIMENSIONAL GEOMETRY

SECTION- A (MCQs) (1 MARK)

Q1.If the x-coordinate of a point P on the line join of Q(2,2,1) and R(5,1,-2) is 4.Then find its z-coordinate.

Q2.Write the direction cosine of a line parallel to z-axis.

Q3. Find the value of λ so that the following lines are perpendicular to each Other . $\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$, $\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$

Q4. If a line makes angle 90° and 60° respectively with the positive direction of x and y axis find the angle which it makes with positive direction of z-axis.

5. Find the angle between the pair of lines given by

$\vec{r} = 3\hat{\imath} + 2\hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})$ and $\vec{r} = 5\hat{\imath} - 2\hat{\jmath} + \mu(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$

Q6.Write the direction cosine of a line equally inclined to the co-ordinate axis.

Q7.Write the vector equation of the following line

$\frac{x-5}{3} = \frac{y+4}{7} = \frac{6-z}{2}$

Q8.Write the cartesian equation of the following line given in vector form.

$\vec{r} = 2\hat{\imath} - \hat{\jmath} - 4\hat{k} + \lambda(\hat{\imath} - \hat{\jmath} - \hat{k})$

Q9. Find the direction cosines of the line passing through the two point (-2,4,-5) and (1,2,3).

Q10. Write the equation of the line passing through the points (-1,2,1) and (3,4,1).

Q11 If a unit vector $\hat{a}$ makes angle $\frac{\pi}{3}$ with $\hat{\imath}$,$\frac{\pi}{4}$ with $\hat{\jmath}$ and an acute angle γ with $\hat{k}$,find the value of γ .

Q12If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{4-z}{k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{-2}$ are mutually perpendicular then find value of k.

Q13. If a line makes angles $\alpha , \beta \text{ and } \gamma$ with the co-ordinate axes. Find the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$.

Q14. If ($\frac{1}{2}$,$\frac{1}{3}$,n) are the direction cosines of a line then find value of n.

Q15. Find the direction cosine of x ,y ,and z axis.

Q16.The cartesian equation of a line are 6x- 2 = 3y+1 = 2z +2. Find d-ratios of the line.

Q17.The equation of straight line passing through the point (a ,b ,c) and parallel to z-axis.

Q18. Find the equation of the straight line through(1,-2,3) and equally inclined To the axes.

Q19. Find angle between the lines $\frac{x}{2} = \frac{y}{2} = \frac{z}{-1}$ and $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{2}$.

Q20.Find the angle between the following pairs of lines .Aline with direction ratios 2,2,2 and a line joining (3,1,4) and (7,2,12).

Assertion reason based questions:-

Directions: In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R).Mark the correct choice as

(A)Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true and R is NOT the correct explanation of A.

(C)A is true but R is false.

(D)A is false but R is True.

Q21.Assertion(A):If two lines are in the same plane i.e they are coplanar, they will intersect each other if they are non-parallel. Hence the shortest distance between them is zero.

If the lines are parallel then the shortest distance between them will be the perpendicular distance between the lines i.e the length of the perpendicular drawn from a point on one line onto the other line.

Reason(R):The angle between the lines with direction ratio(a_1,b_1,c_1) and (a_2,b_2,c_2) is given by $\cos\theta = \frac{a1a2+b1b2+c1c2}{\sqrt{a1^2+b1^2+c1^2}\ \sqrt{a2^2+b2^2+c2^2}}$.

Q22.Assertion(A) : The angle between the straight lines $\frac{x+1}{2} = \frac{y-2}{5} = \frac{z+3}{4}$ and

$\frac{x-1}{1} + \frac{y+2}{2} = \frac{z-3}{-3}$ is 90° .

Reason(R): Skew lines are lines in different planes which are parallel and intersecting.

Q23.Assertion(A) P is a point on the line segment joining the points(3,2,-1) and (6,-4,-2).If x-coordinate of P is 5, then its y coordinate is -2.

Reason(R):The two lines x=ay+b ,z=cy+d and x=a' y+ b' ,z=c'y+d, will be perpendicular iff aa' +bb' +cc' = 0.

Q24.Assertion(A): Direction cosine of a line are the sines of the angles made by the line with the negative directions of the coordinate axes.

Reason(R):The acute angle between the lines x -2 = 0 and $\sqrt{3x - y - 2}$ is 30°.

SECTION-B (VERY SHORT ANSWER) (2 MARKS)

Q1.Find the equation of the line through (1,-1,2) and parallel to the line joining

(-1,0,1) and (2,3,-1).

Q2.Find the value of λ so that the following lines are perpendicular to each

Other $\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$.

Q3.If a line makes angle $\alpha, \beta \ and \ \gamma$ with the co-ordinate axes, then prove that

Cos2α + cos2β + cos 2γ = -1

Q4. Show that the line through the points (1,-1,2),(3,4,-2) is perpendicular to the line through the points (0,3,2) and (3,5,6).

Q5. Show that the line through the points (4,7,8) (2,3,4) is parallel to line through the points (-1,-2,1),(1,2,5).

Q6.Find the equation of the line in vector and cartesian form that passes through the point with position vector $2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and is in the direction $\hat{\imath}+2\hat{\jmath}-\hat{k}$.

Q7.Find the equation of the perpendicular drawn from the point P(2,4,-1) to the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

Q8. Find the vector equation of a line joining the points with position vectors

$\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$,and parallel to the line joining the points with position vectors

$2\hat{\imath} + \hat{\jmath} + 2\hat{k}$.Also find the cartesian equation of this equation.

SECTION-C (SHORT ANSWER) (3 MARKS)

Q1.Find the shortest distance between lines $\hat{r} = (1+\lambda)\hat{\imath} + (2-\lambda)\hat{\jmath} + (\lambda + 1)\hat{k}$ and

$\hat{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu\,(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$.

Q2. Find the points on the line $\frac{x+2}{3} = \frac{y+1}{2} = \frac{z-3}{2}$ at a distance 5 units from the point P(1,3,3).

Q3.Find the vector and cartesian equation of the line passing through the point

(1,2,-4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

Q4.Find the co-ordinate of the foot of perpendicular and the length of the perpendicular drawn from the point P(5,4,2) to the line

$\hat{r} = (-\hat{\imath} + 3\hat{\jmath} + \widehat{k)} + \lambda\,(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$.

Q5.Find the image of the point (1,6,3) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$.

Q6.Find the angle between the lines whose direction cosines are given by the equations 3l + m+ 5n and 6mn – 2nl +5lm = 0 .

Q7. Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Find point of intersection.

Q8.Find the foot of perpendicular drawn from the point (4,2,3) to the line joining (1,-2,3) and (1,1,0).

SECTION-D (LONG ANSWER) (5 MARKS)

Q1.Find the value of β ($\neq 0$),if the length of perpendicular drawn from the point ($\beta, 0, \beta$) to the line $\frac{x}{1} = \frac{y-1}{0} = \frac{z+1}{-1}$ is $\sqrt{\frac{3}{2}}$.

Q2. Find the value of a + b + c where (a,b,c) is the image of (1,2,-3) in the line

$\frac{x+1}{2} = \frac{y-3}{-2} = \frac{z}{-1}$.

Q3.If a point R(4,y,z) lies on the line segment joining the point P(2,-3,4) and

Q(8,0,10) .Find the distance of R from origin.

Q4.The vertices of B and C of a ΔABC lies on the line $\frac{x+2}{3} = \frac{y-1}{0} = \frac{z}{4}$,such that BC= 5 units.If the point A(1,-1,2) then find the area of the triangle ABC.

Q5.Aline makes angle α , β , γ , δ with the four diagonals of a cube ,prove that $\text{Cos}^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$.

Q6.Find the equation of the line passing through (1,-1,1) and perpendicular to the line joining the points (4,3,2),(1,-1,0) and (1,2,-1) (2,1,1).

SECTION-E , (CASE STUDY BASED QUESTIONS) (4MARKS)

Q1. The equation of motion of a rocket are x =2t , y = -4t ,z = 4t ,where the time t is given in seconds , and the distance measured in kilometers

Based on the above information answer the following:-

(i)What is the path of the rocket?

(a)Straight line (b) Circle (c) Parabola (d) none of these (ii)Which is the following points lie on the path of the rocket?

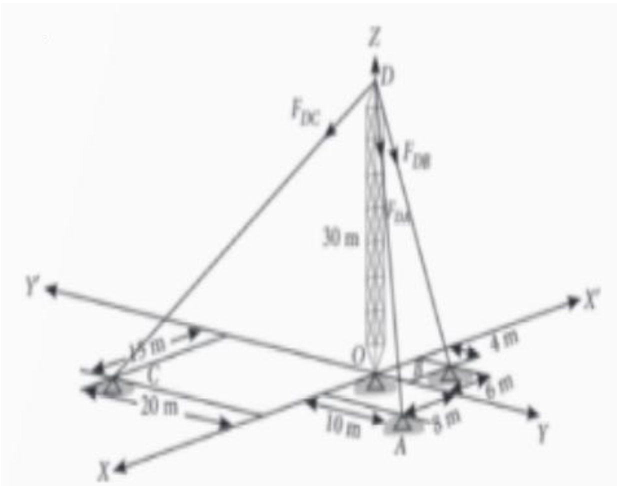
(a) (0,1,2) (b)(0,0,0) (c) (2,-2,2) (d) none of these

(iii)At what distance will the rocket be from the starting point (0,0,0) in 10 second?

(a) 40 Km (b) 60 Km (c) 30 Km (d) 80Km

(iv) If the position of rocket at certain instant of time (3,-6,6) then what will be the height of the rocket from the ground ,which is along the xy- plane .

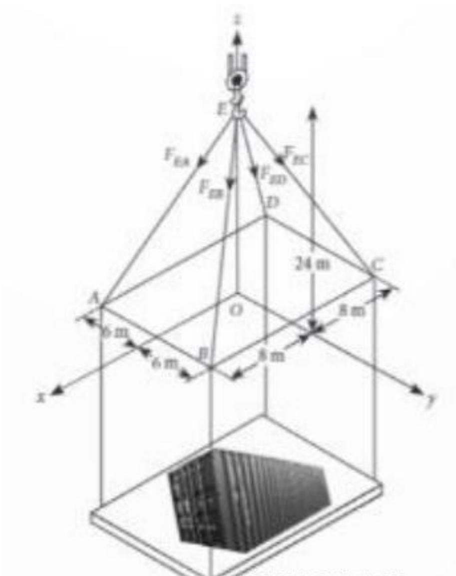
Q2 Based on the given diagram ,answer the following question:-



(i) Find the equation of the line along AD.

(ii) Find the length of DC.

Q3.Based on the information answer the following questions:



50 MINUTES

(i)What is the cartesian equation of line along EA?

(ii)Find the vector equation of the vector $\overrightarrow{ED}$.

Q4.Two motorcycles A and B are running at the speed more than allowed

Speed $\vec{r}= \lambda(\hat{\imath} +2\hat{\jmath} -\hat{k})$ and $\vec{r} = 3\hat{\imath} + 3\hat{\jmath} + \mu(2\hat{\imath} +\hat{\jmath} +\hat{k})$ respectively

Based on the above information ,answer the following questions:-

(i) The cartesian equation of the line along which motorcycle A is running

(a)$\frac{x+1}{1} = \frac{y+1}{2} = \frac{z-1}{-1}$ (b)$\frac{x}{1} = \frac{y}{2} = \frac{z}{-1}$ (c)$\frac{x}{1} = \frac{y}{2} = \frac{z}{1}$ (d)none of these

(ii)The direction cosines of line along which motorcycle A is running

(a)$< 1,-2,1 >$ $(b) < 1,2,-1 >$ $(c) < \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}} >$ $(d) \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} >$

(iii)The shortest distance between the given lines

(a)4 units (b) $2\sqrt{3}$ units (c) $3\sqrt{2}$ units (d) 0 units

(iv)The motorcycles will meet with an accident at the point

(a)(-1,1,2) (b)(2,1,-1) (c)(1,2,-1) (d) does not exist

Chapter 12: LINEAR PROGRAMMING

MCQs

1. In an LPP, if the objective function Z = px + qy has same maximum at two corner points of the feasible region, then the number of points at which maximum value of Z occurs is

   a) 0 b) 1 c) 2 d) infinite

   Solution: every point on the line joining these two corner points gives same maximum.

   Ans: d

2. The objective function Z = ax + by of an LPP has maximum value 42 at (4, 6) and minimum value 19 at (3, 2). Which of the following is true

   a) a = 9, b = 1 b) a = 5, b = 2 c) a = 3, b = 5 d) a = 5, b = 3

   solution: Z(4, 6) = 42 => 4a + 6b =42 Z(3, 2) = 19 => 3a + 2b = 19 solving we get

   a = 3, b = 5

   Ans: c

3. The corner points of the feasible region of a linear programming problem are (0, 4), (8, 0) and (20/3, 4/3). If Z = 30x + 20y is the objective function, then (Maximum value of Z – Minimum value of Z) is equal to

   a) 40 b) 96 c) 160 d) 136

   Solution: Max. = Z(8, 0) =240 and Min. = Z(0, 4) = 80

   Ans: c

4. If the corner points of the feasible region of an LPP are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5), then the minimum value of the objective function Z = 4x + 6y occurs at

   a) (0, 2) only b) (3, 0) only

   c) The midpoint of the line segment joining the points (0, 2) and (3, 0) only

   d) Every point on the line segment joining the points (0, 2) and (3, 0)

   Solution: If Z has same min.value at two points, then Z has same min. value at every point on the line segment joining the two points.

   Ans: d

ASSERTION AND REASONING QUESTIONS

The following questions consists of two statements-Assertion(A) and Reason(R).

Answer these questions selecting appropriate option given below.

a) Both A and R are true and R is the correct explanation for A

b) Both A and R are true and R is not the correct explanation for A

c) A is true but R is false

d) A is false but R is true

Assertion (A):The max. value of Z = x + 3y subject to 2x + y ≤ 20, x + 2y ≤ 20

x ≥ 0, y ≥ 0 is 30

Reason(R): The variables that are present in the problem are called decision variables.

Solution: corner points are (0, 0), (10, 0), (20/3, 20/3) and (0, 10)

Zmax. =x + 3y = 0 + 3x10 = 30 both A and R are true but R is not the correct explanation for A

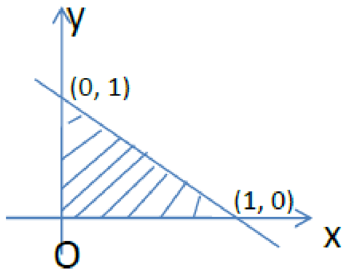
Ans: b

Short answer type questions(2 marks)

1. Find $Z_{max.}$=3x + 2y subject to the constraints x + y ≤ 2, x, y ≥0

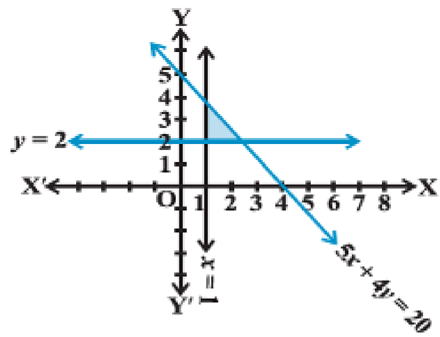
Solution: Corner points (0, 0), (2, 0), (0, 2)

Corner points	Z = 3x + 2y
(0, 0)	0
(2, 0)	6 Max.
(0, 2)	4



2. Write the linear inequations for which the shaded area in the following figure is the solution set.

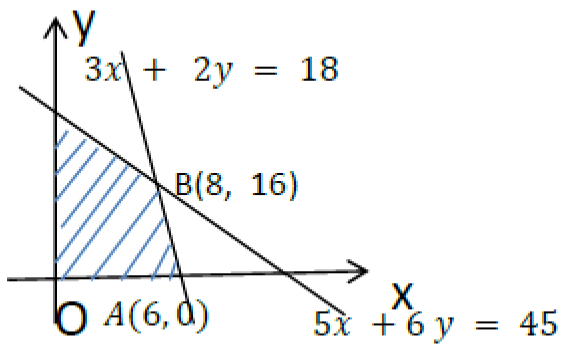
Solution: 5x + 4y ≤ 20, x ≥ 1, y ≥ 2



Short answer type questions(3 marks)

1. Maximise : Z = 60x + 40y subject to 5x + 6y ≤ 45, 3x + 2y ≤ 18 x, y ≥ 0

Solution: On solving 5x + 6y = 45, 3x + 2y = 18 we get (9/4, 45/8)

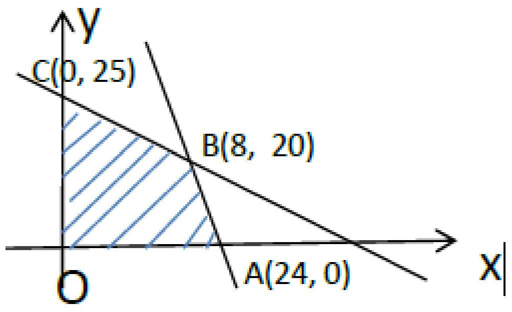


Corner points	Z =60x + 40 y
(0, 0)	0
(0, 15/2)	300
(9/4, 45/8)	360 Maximum
(6, 0)	360Maximum

Maximum value of Z is 360 at any point on the line segment joining (6, 0) and (9/4, 45/8)

2. Maximise Z = 100x + 120y subject to the constraints 5x + 8y ≤ 200, 5x + 4y ≤120, x,y≥0

Solution:



Corner points	Z = 100x + 120y
(0, 0)	0
(0, 25)	3000
(24, 0)	2400
(8, 20)	3200

Maximum value of Z is 3200 at point (8, 20).

Questions for Practice

1. If the objective function . is Z = 5x + 7 and the corner points of the bounded feasible region are (0, 0), (7, 0) ,(3 (3, 4) and (0, 2), then the maximum value of Z occurs at

(a) (0,0) (b) (7,0) (c) (3,4) (d) (0, 2)

2.If the corner points of the feasible region for an L.P.P are (0, 2), (3, 0), (6, 0) ,(6, 8) and (0, 5), then the minimum value of the objective function F= 4x + 6y occurs at

(a) (0, 2) only (b) (3,0) only

(c) the mid-point of the line segment joining the points (0, 2) and (3, 0) only

(d) every point on the line segment joining the points (0, 2) and (3, 0).

3. The corner points of the feasible region for an L.P.P. are (0, 10), (5, 5), (15, 15) and (0, 20). If the objective function is Z = px + qy p, q > 0 then the condition on p and q so that the maximum of Z occurs at (15, 15) and (0, 20) is

(a) p = q (b) p = 2q (c) q = 2p (d) q = 3p

4.The graph of the inequality 2x + 3y > 6 is

(a) half plane that contains the origin

(b) half plane that neither contains origin nor the points of the line 2x + 3y = 6

(c) whole XOY-plane excluding the points on the line 2x + 3y = 6

(d) entire XOY-plane.

ASSERTION AND REASONING QUESTIONS

The following questions consists of two statements-Assertion(A) and Reason(R).

Answer these questions selecting appropriate option given below.

a) Both A and R are true and R is the correct explanation for A

b) Both A and R are true and R is not the correct explanation for A

c) A is true but R is false

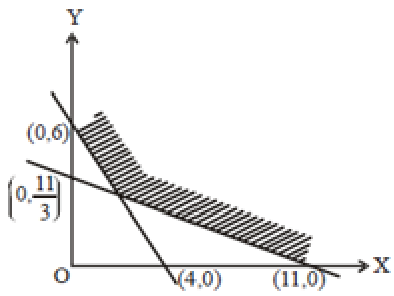
d) A is false but R is true

Assertion (A): If the feasible region for an L.PP. is bounded, then the objective function Z = ax + by has both maximum and minimum values.

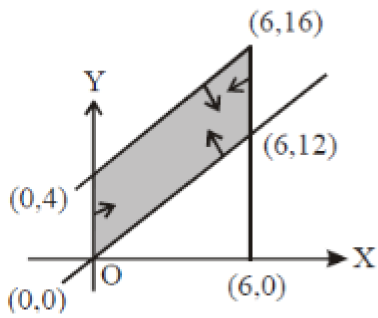
Reason(R): A feasible region of a system of linear inequalities is said to be bounded if it can be enclosed within a circle.

Short answer type questions

1. Maximise Z = 80x + 120y subject to the constraints 3x + 4y ≤ 60, x + 3y ≤ 30, x, ≥ 0.
2. Let Z = ax + by has optimal value at two points (2, 3) and (5, 7), then find the relation between a and b.
3. Find the maximum value of Z = 4x + 3y subject to x + y ≤ 10, x, y ≥ 0.
4. A manufacturing company makes two types of television sets; one is black and white and the other is colour. The company has resources to make at most 300 sets a week. It takes Rs 1800 to make a black and white set and Rs 2700 to make a coloured set. The company can spend not more than Rs 648000 a week to make television sets. If it makes a profit of Rs 510 per black and white set and Rs 675 per coloured set, how many sets of each type should be produced so that the company has maximum profit? Formulate this problem as a LPP given that the objective is to maximise the profit.



5. For the following feasible region, write the linear constraints.
6. The feasible region for LPP is shown shaded in the figure. Let Z = 3 x – 4 y be the objective function, then write the maximum value of Z .



7. Maximise :Z = 6x + 3y subject to 4x + y ≥80, 3x + 2y ≤ 150, x + 5y ≥ 15, x, y ≥ 0.
8. Minimise: Z = 200x + 500y subject to x + 2y ≥ 10, 3x + 4 y ≤ 24, x, y ≥ 0.
9. Minimise : Z =x +2y subject to x + 2y ≥ 100, 2x – y ≤ 0, 2x + y ≤ 200, x, y ≥ 0.
10. Maximise : Z = 300x + 190y subject to x + y ≤ 24, x + ½ y ≤ 16, x, y ≥ 0

1. The objective function of a linear programming problem is:

A. a constraint B. function to be optimised

C. A relation between the variables D. None of these

2. A set of values of decision variables that satisfies the linear constraints and non-negativity conditions of an L.P.P. is called its:

A. Unbounded solution B. Optimum solution

C. Feasible solution D. None of these

3. The maximum value of Z = 3x + 4y subjected to constraints x + y ≤ 4, x ≥ 0 and y ≥ 0 is:

A. 12 B. 14 C. 16 D. None of the above

4. The minimum value of Z = 3x + 5y subjected to constraints x + 3y ≥ 3, x + y ≥ 2, x, y ≥ 0 is:

A. 5 B. 7 C. 10 D. 11

5. The point which does not lie in the half-plane 2x + 3y -12 < 0 is:

A. (2,1) B. (1,2) C. (-2,3) D. (2,3)

6. The optimal value of the objective function is attained at the points:

A. on X-axis B. on Y-axis C. corner points of the feasible region D. none of these

7. Region represented by x ≥ 0, y ≥ 0 is:

A. first quadrant B. second quadrant C. third quadrant D. fourth quadrant

8. Maximise Z = 3x + 4y subject to the constraints: x + y ≤ 4, x ≥ 0, y ≥ 0.

A. Maximum Z = 16 at (0, 4) B. Maximum Z = 19 at (1, 5)

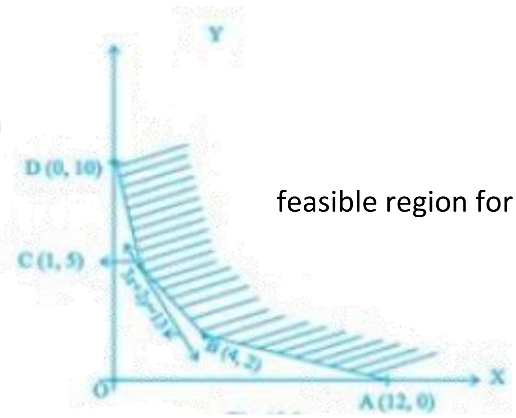
C. Maximum Z = 18 at (1, 4) D. Maximum Z = 17 at (0, 5)

9. Minimise Z = 13x – 15y subject to the constraints:

3*4=12

x + y ≤ 7, 2x – 3y + 6 ≥ 0, x ≥ 0, y ≥ 0.

10. Determine the minimum value of Z = 3x + 2y (if any), if the feasible region for an LPP is shown in Fig.



11. Minimise Z = 13x - 15y, subject to the constraints:

$$x + y \leq 7, 2x - 3y + 6 \geq 0, x \geq 0, y \geq 0.$$

12. Maximise Z = x + y subject to x + 4y ≤ 8, 2x + 3y ≤ 12, 3x + y ≤ 9, x ≥ 0, y ≥ 0.

Answers

1: B. function to be optimized 2: C. Feasible solution 3: C. 16 4: B. 7

5: D. (2,3) 6: C. corner points of the feasible region 7: A. first quadrant 8: A

9: Z is min. at C(0, 2) and Zmin.= 13x0 -15x2 = 30

10. the open half plane determined by $3x + 2y < 13$ and R do not have a common point. So, the smallest value 13 is the minimum value of Z.

11. Corner points are O(0, 0), B(7, 0), E(3, 4) and C(0, 2), the minimum value is -30 at the point (0, 2)

12. the maximum value is 43/11 at (28/11 , 15/11)

TEST -2

M.M.: 30 Marks

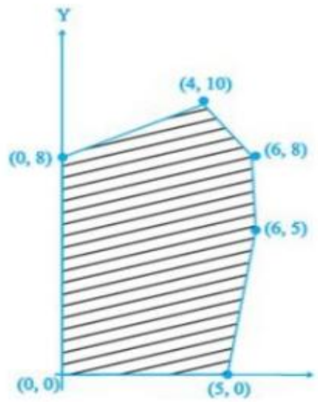
1. A linear programming problem is one that is concerned with

a. finding the optimal value (maximum or minimum) of a linear function of several variables

b. finding the limiting values of a linear function of several variables

c. finding the lower limit of a linear function of several variables

d. finding the upper limits of a linear function of several variables



2.The feasible solution for an LPP is shown in Figure.

Let $Z = 3x – 4y$ be the objective function. Minimum of Z occurs at

a. (0, 8) b. (0, 0) c. (5, 0)d. (4, 10)

3. The corner points of the feasible region for an LP.P. are (0, 3), (1, 1) and (3, 0). If objective function is $z = px + qy$ $p , q > 0$, then the condition on pand q so that the minimum of Z occurs at (3, 0) and (1, 1) is

(a) $p = 2q$ (b) $p = q/2$ (c) $p = 3q$ (d) $p = q$

4. A book publisher sells a hard cover edition of a book for 272 and a paperback edition for ₹40. In addition to a fixed weekly cost of ₹9600, the cost of printing hard cover and paperback editions are 56 and 28 per book respectively. Each edition requires 5 minutes on the printing machine whereas hardcover binding takes 10 minutes and paperback takes 2 minutes on the binding machine. The printing machine and the binding machine are available for 80 hours each week. Formulate the linear programming problem to maximize the publisher's profit.

5. Jimmy is a young biker. He finds that if he rides his bike at 25 km/hr., he has to spend Rs. 2 per km on petrol. If he increases his speed to 40 km/hr. his petrol cost also increases to Rs 5 per km. Jimmy decides to spend Rs 100 on petrol cost but he also wants to travel the maximum distance within one hour. Express this problem as L.L.P.

6. Solve the Linear Programming Problem graphically.

Maximise and minimise: $z = 3x + 9y$.

Subject to constraints $x + 3y \le 60$ $x + y \ge 10$, $x \le y$, $x \ge 0$; $y \ge 0$.

7. Find graphically the minimum value of $z = -50x + 20y$, subject to the constraints $2x-y \geqslant -5$, $3x+y \geqslant 3$, $2x-3y \leqslant 12$, $x \geqslant 0$, $y \geqslant 0$.

8. Solve the Linear Programming Problem graphically.

Maximise and minimise: $z = 60x + 15y$.

Subject to constraints $x + y \leq 50$ $3x + y \leq 90$, $x \geq 0$; $y \geq 0$.

9. Find graphically the maximum value of $z = 5x + 2y$, subject to the constraints $3x+5y \leq 15$, $5x+2y \leq 10$, $x \geqslant 0$, $y \geqslant 0$.

10. Find graphically, the maximum value of $Z = 2x + 5y$.

Subject to constraints $2x + 4y \leq 8$ $3x + y \leq 6$, $x + y \leq 4$ $x \geq 0$; $y \geq 0$

5. Maximise and minimise: $z = 5x + 10y$,subject to constraints $x + 2y \leq 120$, $x + y \geq 60$, $x \geq 0$; $y \geq 0$.
6. Minimize and maximize $Z = x + 21$ subject to the constraints $x + 2y \geqslant 100$, $2x - y \leqslant 0$, $2x + y \leqslant 200$, $x \geqslant 0$, $y \geqslant 0$

ANSWERS

1. a 2. a. (0, 8)

3. B)

4. Maximize profit $Z = 16x + 12y - 9600$ Subject to the constraints $x + y \leqslant 960$, $5x + y \leqslant 2400$, $x, y \geqslant 0$.

5.Jimmy drives x km at a speed of 25 km/hr. and y km at 40 km/hr. speed.

$2x + 5y \leq 100$,$40x + 25y \leq 1000$,$x \geq 0$, $y \geq 0$.

6. Z= 180 is maximum at all the points that join (15, 15) and (0, 20).

7.Z does not have any minimum value.

8.Minimum of Z is 0 at (0,0) and max value is 1800 at (30,0)

9. Z is max at both (2,0) and (20/19, 45/19) . Thus we have multiple optimal solutions.

10.Max value 10 at (0,2)

11.min value 300 at (60,0) max value 600 on line segment joining(120,0) and (60,30).

12. max value 400 at (0,200) min value 100 on line segment joining(20,40) and (0,50).

Chapter 13: PROBABILITY

IMPORTANT QUESTIONS:

LEVEL 1 (MCQ/ONE MARK QUESTIONS)

1 Probability that A speaks truth is 4/ 5 . A coin is tossed. A reports that a head appears. The probability that actually there was head is

(A) 4/ 5 (B) 1/ 2 (C) 1/ 5 (D) 2/ 5

2 The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

(A) 1 (B)2 (C) 5 (D) 8/ 3

3 Find P(E|F), where E: no tail appears, F: no head appears when two coins are tossed in the air.

(a) 0 (b) ½ (c) 1 (d) None of the above

4 If A and B are two independent events, then the probability of occurrence of at least one of A and B is given by:

(a) 1+ P(A′) P (B′) (b) 1− P(A′) P (B′)

(c) 1− P(A′) + P (B′) (d) 1− P(A′) – P (B′)

5 If E and F are independent events, then;

(a) P(E ∩ F) = P(E)/ P(F) (b) P(E ∩ F) = P(E) + P(F)

(c) P(E ∩ F) = P(E) . P(F) (d) None of the above

6 A police officer fires three bullets at a thief. The probability that the thief will be killed by one bullet is 0.8. Find the probability of the thief being still alive?

(a) 0.008 (b) 0.0016 (c) 0.64 (d) None of the above

7 If P(A) = 4/5 and P(AB) = 7/10, then P(B/A) is equal to

(a) 1/10 (b) 1/8 (c) 7/8 (d) 17/20

8 The events E and F are independent. If P(E) = 0.3 and P(E U F) = 0.5, then P(E/F) – P(F/E) equals

(a) 1/7 (b) 2/7 (c) 3/35 (d)1/70

9 If A and B be independent events with P(A) = 1/4 and P(A U B) = 2 P(B) – P(A). Find P(B)

(a) 1/4 (b) 3/5 (c) 2/3 (d) 2/5

10 The probability that A speaks the truth is 4/5 and that of B speaking the truth is ¾ . The probability that they contradict each other in stating the same fact is

(a) 7/20 (b) 1/5 (c) 3/20 (d) 4/5

11 If E and F are independent events, then find $P(E \cap F^{'})$.

12 If E and F are independent events, then find $P(E' \cap F^{'})$.

13 If A and B are two events such that P(A) + P(B) – P(A and B) = P(A), then find P(A/B).

14 If A and B are two events, given that A is a subset of B and P(A) is non zero, then find P(A/B).

15 Let X denotes the number of hours you study on a Sunday. Also it is known that

$$P(X=x)=\begin{cases}0.1 & if\ x=0\\ kx & if\ x=1\ or\ 2\\ 0, & otherwise\end{cases}$$ where k is a constant.

What is the probability that you study at least two hours.

LEVEL 2 (TWO/THREE MARKS QUESTIONS)

1 A die is tossed thrice. Find the probability of getting an odd number at least once

2 An unbiased die is thrown twice. Let the event A be 'odd number on the first throw' and B the event 'odd number on the second throw'. Check the independence of the events A and B.

3 A person has undertaken a construction job. The probabilities are 0.65 that there will be strike, 0.80 that the construction job will be completed on time if there is no strike, and 0.32 that the construction job will be completed on time if there is a strike. Determine the probability that the construction job will be completed on time.

4 Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as (i) number greater than 4 (ii) six appears on at least one die

5 From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs

6 Probability of solving specific problem independently by A and B are 1/ 2 and 1/ 7 respectively. If both try to solve the problem independently, find the probability that (i) the problem is solved (ii) exactly one of them solves the problem.

7 Three cards are drawn successively, without replacement from a pack of 52 well shuffled cards. What is the probability that first two cards are kings and the third card drawn is an ace?

8 In a hostel, 60% of the students read Hindi news paper, 40% read English news paper and 20% read both Hindi and English news papers. A student is selected at random. (a) Find the probability that she reads neither Hindi nor English news papers. (b) If she reads Hindi news paper, find the probability that she reads English news paper. (c) If she reads English news paper, find the probability that she reads Hindi news paper.

9 Events A and B are such that P (A) = 1/ 2 , P(B) = 7/ 12 and P(not A or not B) = 1/ 4 . State whether A and B are independent ?

10 A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2 + k$

Determine (i) k (ii) P(X < 3) (iii) P(X > 6)

LEVEL 3 FOUR/FIVE MARKS QUESTIONS:

1 Of the students in a college, it is known that 60% reside in hostel and 40% are day scholars (not residing in hostel). Previous year results report that 30% of all students who reside in hostel attain A grade and 20% of day scholars attain A grade in their annual examination. At the end of the year, one student is chosen at random from the college and he has an A grade, what is the probability that the student is a hostlier?

2 In answering a question on a multiple choice test, a student either knows the answer or guesses. Let 3 4 be the probability that he knows the answer and 1 4 be the probability that he guesses. Assuming that a student who guesses at the answer will be correct with probability 1 4 . What is the probability that the student knows the answer given that he answered it correctly?

3 In a factory which manufactures bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 5, 4 and 2 percent are respectively defective bolts. A bolt is drawn at random from the product and is found to be defective. What is the probability that it is manufactured by the machine B?

4 Given three identical boxes I, II and III, each containing two coins. In box I, both coins are gold coins, in box II, both are silver coins and in the box III, there is one gold and one silver coin. A person chooses a box at random and takes out a coin. If the coin is of gold, what is the probability that the other coin in the box is also of gold?

5 A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively 3/10, 1/5, 1/10 and 2/5 . The probabilities that he will be late are ¼, 1/3 and 1/12, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

6 CASE STUDY 1:

A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots From this situation answer the following: 1. Let the target is hit by A, B: the target is hit by B and, C: the target is hit by A and C. Then,

(i) the probability that A, B and, C all will hit, is

a. 4/5 b. 3/5 c. 2/5 d. 1/5

(ii), what is the probability that B, C will hit and A will lose?

a. 1/10 b. 3/10 c. 7/10 d. 4/10 3.

(iii) With reference to the events mentioned in (i), what is the probability that 'any two of A, B and C will hit?

1. 1/30 2. 11/30 3. 17/30 4. 13/30 4.

(iv) What is the probability that 'none of them will hit the target'?

a. 1/30 b. 1/60 c. 1/15 d. 2/15 5.

(v) What is the probability that at least one of A, B or C will hit the target?

a. 59/60 b. 2/5 c. 3/5 d. 1/60

7 Case Study 2

The reliability of a COVID PCR test is specified as follows: Of people having COVID, 90% of the test detects the disease but 10% goes undetected. Of people free of COVID, 99% of the test is judged COVID negative but 1% are diagnosed as showing COVID positive. From a large population of which only 0.1% have COVID, one person is selected at random, given the COVID PCR test, and the pathologist reports him/her as COVID positive. Based on the above information, answer the following

1. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually having COVID?

a. 0.001 b. 0.1 c. 0.8 d. 0.9

2. What is the probability of the 'person to be tested as COVID positive' given that 'he is actually not having COVID'?

a. 0.01 b. 0.99 c. 0.1 d. 0.001

3. What is the probability that the 'person is actually not having COVID?

a. 0.998 b. 0.999 c. 0.001 d. 0.111

4. What is the probability that the 'person is actually having COVID given that 'he is tested as COVID positive'?

a. 0.83 b. 0.0803 c. 0.083 d. 0.089

5. What is the probability that the 'person selected will be diagnosed as COVID positive'?

a. 0.1089 b. 0.01089 c. 0.0189 d. 0.189

8 CASE STUDY 3

In answering a question on a multiple choice test for class XII, a student either knows the answer or guesses. Let 3/5 be the probability that he knows the answer and 2/5 be the probability that he guesses. Assume that a student who guesses at the answer will be correct with probability 1/3. Let E1, E2, E be the events that the student knows the answer, guesses the answer and answers correctly respectively. Based on the above information, answer the following

1. What is the value of P(E1)?

a. 2/5 b. 1/3 c. 1 d. 3/5

2. Value of P(E | E1) is

a. 1/3 b. 1 c. 2/3 d. 415

3. ∑ P(E|Ek) P(Ek) k=2 k=1 Equals

a. 11/15 b. 4/15 c. 1/5 d. 1

4. Value of $\sum P(E_k)$ $k=2$ $k=1$

a. 1/3 b. 1/5 c. 1 d. 3/5

5. What is the probability that the student knows the answer given that he answered it correctly?

a. 2/11 b. 5/3 c. 9/11 d. 13/3

ANSWERS:

LEVEL 1(ONE MARK QUESTIONS)

1.a 2. b 3. a 4. b 5. c 6. a 7. c 8. b 9. d 10. a

11.$P(E \cap F') = P(E).P(F')$ 12.$P(E' \cap F') = P(E').P(F')$ 13. 1 14. 1 15. 0.6

LEVEL 2 (TWO/THREE MARKS QUESTIONS)

1. 7/8 2. A and B are independent events 3. 0.488

4.

X	0	1	2
P(X)	4/9	4/9	1/9

X	0	1
P(X)	25/36	11/36

5.

X	0	1	2	3	4
P(X)	256/625	256/625	96/625	16/625	1/625

6. 4/7, ½ 7. 2/5225 8. (a)1/5, (b) 1/3, (c) ½

9. A and B are NOT independent events

10. (i)1/10,(ii) 3/10, (iii) 17/100

LEVEL 3 (FOUR/FIVE MARKS QUESTIONS)

1. 9/13 2. 12/13 3. 28/69 4. 2/3 5. 1/2

CASE STUDY 1

1. (c) 2/5 2. (a) 1/10 3. (d) 13/30 4. (b) 1/60 5. (a) 59/60

CASE STUDY 2

1. (d) 0.9 2. (a) 0.01 3. (b) 0.999 4. (c) 0.083 5. (b) 0.0108

CASE STUDY 3

1. (d) 3/5 2. (b) 1 3. (a) 11/15 4. (c) 1 5. (c) 9/11